AP Physics 1 Review

AP-Physics 1: Problem Solving Tools

1. **Newton's Laws of Motion (Kinematics)**
2. **Work - Kinetic Energy Theorem (Open System)**
3. **Impulse - Momentum Theorem (Open System)**
4. **Conservation of Energy (Closed System)**
5. **Conservation of Momentum (Closed System)**

Types of Error (in labs)

1. **System Error**

Materials, Air Resistance, Friction

2. **Mathematical Error**

Truncating, Calculation error

3. **Observational Error**

Parallax, Reaction time, measuring distance

UNIT 1: 1-D Motion

Fundamentals of Unit 1 Physics

Motion in one direction, x-axis or y-axis

Position (m) : location of an object in reference to a stationary origin

- Vector quantity = has magnitude and direction

Displacement (m) : the net change in position of an object

- Vector quantity = has magnitude and direction
- $\Delta x = x_f - x_o$

Distance (m) : amount traveled to get to the final position

- Scalar quantity = has magnitude and no direction
- Always positive
- Determinant of how you got to the destination

Speed (m/s) : how fast an object is traveling in a given time

- The speed at a specific instance in time

Average Speed (m/s) : total distance traveled in a total amount of time

- Average speed in a chunk of time

Velocity (m/s) : how fast an object is traveling in a given time in a certain direction

Average velocity (m/s) : total displacement traveled in a total amount of time

- $v = \Delta x / \Delta t$
- Averaging speeds is NOT equal to average speed
  - 400m / 75 s = 5.3 m/s
  - 400m / 50 s = 8 m/s
  - 800m / 125s = 6.4 m/s
  - WRONG = (5.3 + 8) / 2 = 6.7 m/s

Acceleration (m/s^2) : how fast the velocity is changing. Rate of change in velocity. Change in velocity per unit of time.

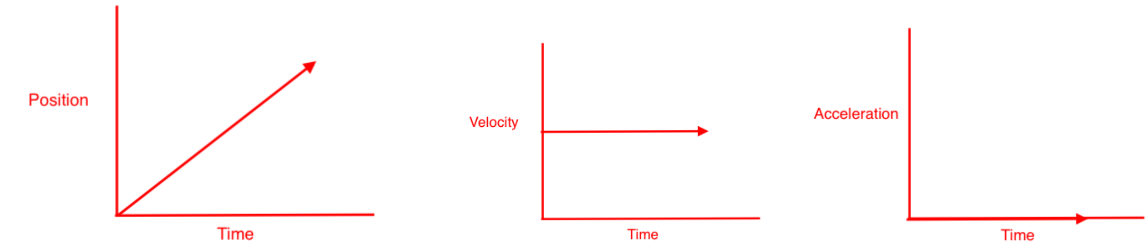
- Vector quantity = has magnitude and direction
- $a = \Delta v / \Delta t = v_f - v_o / t_f - t_o$
- Speed up and slowing down is both accelerating, NOT deceleration

Percent Error (%) : margin of error between the data gained from the experiment or the actual value compared to the expected value

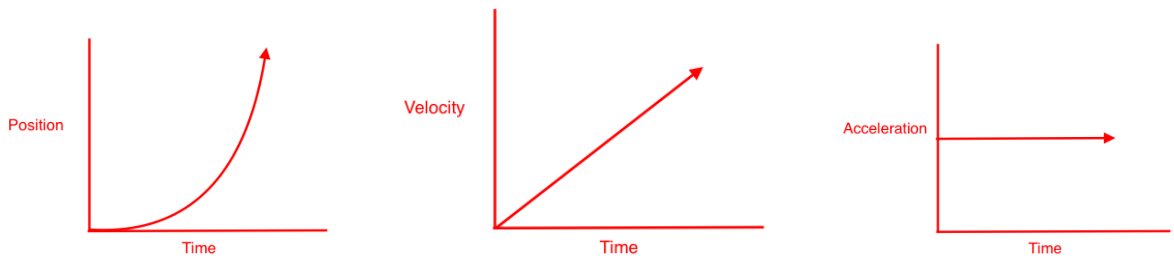
- % Error = $\left|\frac{Actual\ Value - Expected\ Value}{Expected\ Value}\right| * 100\%$

__Types of Uniform Motion__

Uniform Motion (UM) : motion where objects move with non zero constant velocity

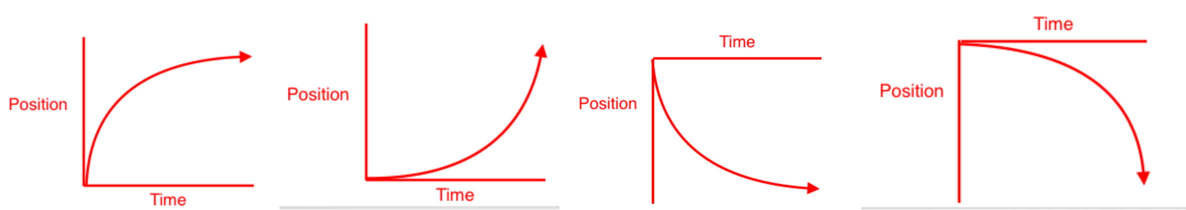


Uniform Accelerated Motion (UAM) : motion where objects move with non zero constant acceleration



Motion Graphs:

- Slope on position vs time graph = velocity
- Slope on velocity vs time graph = acceleration
- Area under the curve on velocity vs. time graph = displacement



Slowing down in positive direction | Speeding up in positive direction | Slowing down in negative direction | Speeding up in negative direction

Mathematical Model: graph, table, and equation

__Kinematics__

Kinematic Equations: can only be used when objects are under a non-zero constant acceleration

- Recognize when to use kinematics, usually when given time, velocity, initial velocity, or distance.

Variables used:

1. Δx displacement [m]
2. V_f final velocity [m/s]
3. V_o initial velocity [m/s]

4. a acceleration [m/s^2]
5. t time [s]

Equations and missing variables: (first three are most important)

$V = V_o + at$	Δx
$\Delta x = V_o t + \frac{1}{2} at^2$	V_f
$V_f^2 = V_o^2 + 2a\Delta x$	t
$\Delta x = t/2\ (V_f + V_o)$	a
$\Delta x = V_f t - \frac{1}{2} at^2$	V_o

Freefall

Freefall: a motion of an object solely under the influence of gravitational acceleration

- Even if an object is traveling upwards, they are still in freefall
- Even if an object is thrown downwards, they are still in freefall
- Acceleration due to gravity for Earth is -9.8 m/s^2 or -10 m/s^2
- Different signs for velocity compared to acceleration = Slowing down
  - Velocity positive, acceleration negative and vice versa
- Same signs for velocity compared to acceleration = Speeding up
  - Velocity positive, acceleration positive and both negative

Segmented Motion 2.0

Motion of two objects rather than one

- Accelerated motion = use kinematics
- Uniform motion = velocity * time = v * t
- Equate displacements
  - Δx (UAM) = Δx (UM)
  - $\Delta x = V_o t + \frac{1}{2} at^2 = V * t$

TIPS ON HOW TO SOLVE PROBLEMS:

1. When doing segmented motion with two objects, one UM and one UAM, equate variables to eliminate an unknown variable
2. As you go from a position → velocity → acceleration graph, the "powers" of the graph decreases
   a. Uniform velocity
      i. Position = x Velocity = constant (non-zero horizontal line) Acceleration = 0
   b. Uniform acceleration
      i. Position = x^2 Velocity = x Acceleration = constant (non-zero horizontal line)

UNIT 2: 2-D Motion and Vectors

Fundamentals of Unit 2 Physics

Motion in two directions, x-axis and y-axis
Vertical and horizontal directions are independent of each other, simply 1D motion problems

Vectors and Scalars: Quantities that are dependent on magnitude and direction are vector quantities. Quantities that are dependent on only magnitude are scalar quantities.

Scalar:	**Vectors**:
Mass	Displacement
Temperature	Velocity
Volume	Acceleration
Time	Force
Distance	Momentum
Speed	
Magnitude of Acceleration	

Coordinate Systems

Cartesian coordinate system (x,y) : using x and y coordinates

Polar coordinate system (r, θ) : using radius and angles, used to display direction

- θ (theta) represents angle or direction as measured from positive x-axis or to the East

Adding Vectors

Vector Addition: Cartesian coordinate to Polar coordinates
Vector Resolution: Polar coordinate to Cartesian coordinates

As angle between vectors increase, magnitude of resultant vector decreases
Antiparallel: vectors going in opposite directions
Parallel: vectors going in same directions
Largest resultant vector when angle is 0˚
Smallest resultant vector when angle is 180˚

Type	Resultant vector	x-component	y-component
Displacement	Δr	Δx	Δy
Velocity	v	v_x	v_y
Acceleration	a	a_x	a_y

Scaling a Vector (changes magnitude, * scalar factor) : multiplying a vector by a scalar or a number

- v = 4 m/s @ 50˚ from +x-axis
- $2v$ = 8 m/s @ 50˚ from +x-axis

Opposing vectors (changes direction, +180˚) : reversing a vector to go in the opposite direction
- v = 4 m/s @ 50˚ from +x-axis
- $-v$ = 4 m/s @ 230˚ from +x-axis
- $-2v$ = 8 m/s @ 230˚ from +x-axis

Adding vectors that are obtuse (not perpendicular, parallel or antiparallel) : add x-components of all resultant vectors and y-components of all resultant vectors to get total x and y components of the resultant vector, then use trigonometry to obtain resultant vector (hypotenuse) and angle (θ)

<u>Projectile Motion Conceptually</u>

Any two-dimensional motion of an object where horizontal motion is a constant velocity (UM) and vertical motion is a free-fall motion (UAM)

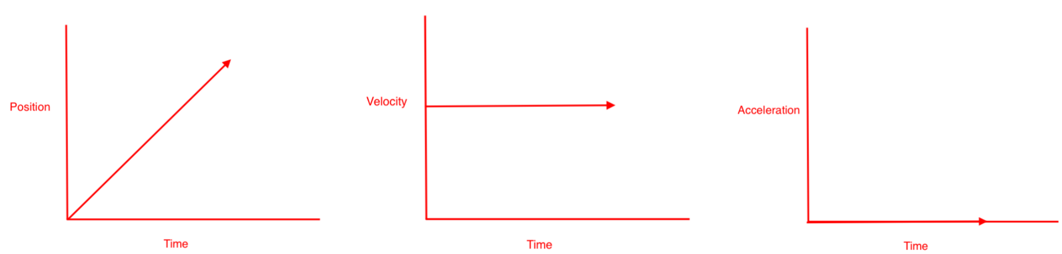
Using two columns, one for horizontal Uniform Motion and one for vertical Uniform Accelerated Motion

Horizontal (UM)	Vertical (UAM or freefall)
V_x = Δx = t =	V_o = V_f = a = Δy = t =
Solved with $\Delta x = v * t$	Solved with kinematics

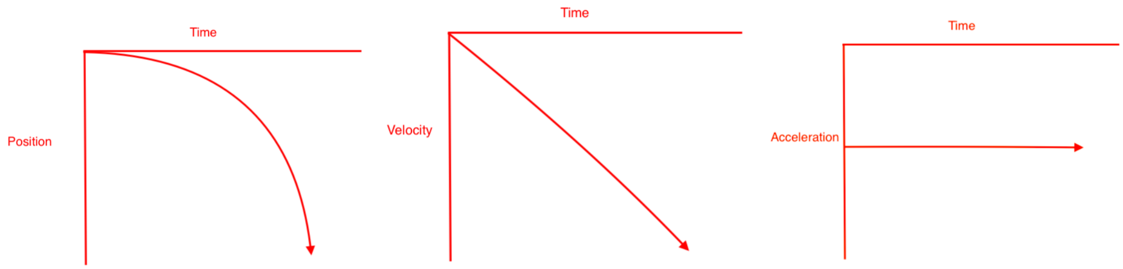
<u>Types of projectile motion:</u>

1. Horizontally launched off cliff
- Using kinematics to solve for range, velocity, time, acceleration

Uniform Motion Graphs (Horizontal) :



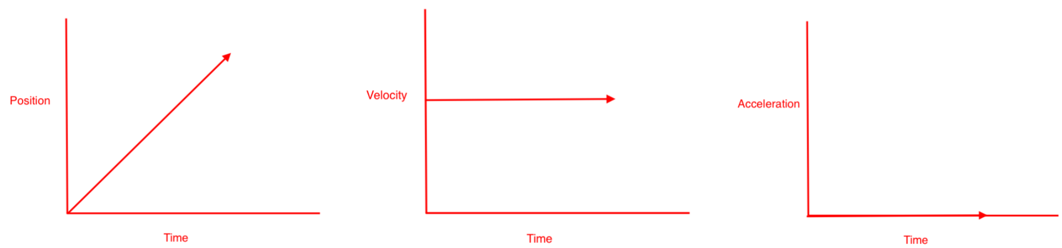
Uniform Acceleration Motion Graphs (Vertical) :



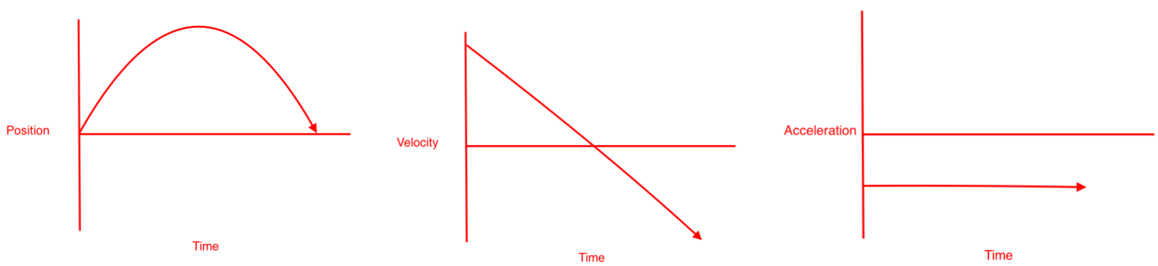
2. Angled projectile from ground

- Max height is reached at ½ time
- Velocity in the y direction at apex is 0 m/s

Uniform Motion Graphs (Horizontal) :

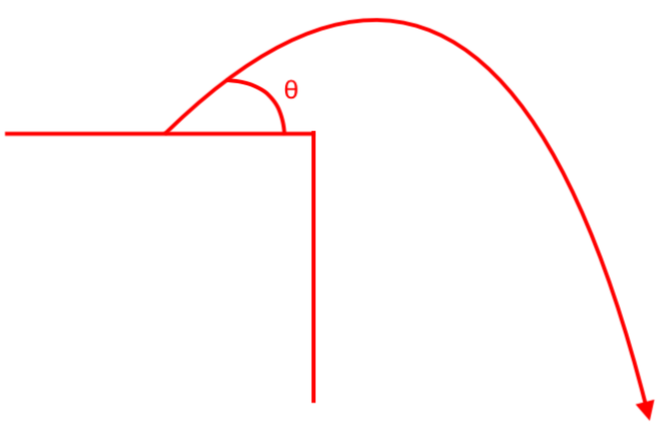


Uniform Acceleration Motion Graphs (Vertical) :



3. Angled projectile off a cliff

- Combines projectile motion off a cliff as well as angled projectile from ground



TIPS ON HOW TO SOLVE PROBLEMS:

1. Add vectors by isolating by direction, x-direction from y-direction and vice versa
2. All directions are from positive x-axis, +x-axis, FPXA (from positive x-axis), East
3. Use two-column approach: horizontal uniform motion and vertical uniform acceleration motion

UNIT 3: Forces and Newton's Laws

Fundamentals of Unit 3 Physics

1 Newton = force to accelerate a 1 kg object at a rate of 1 m/s^2

Forces:

1 - Tension	T or F_T	force in a string
2 - Normal Force	F_N	force applied by a surface (perpendicular)
3 - Weight	F_g or mg	gravitational force
4 - Force of Friction	F_F or F_{fs} or F_{fk} or F_{fmax}	due to roughness of surface (parallel)
5 - Drag or Air resis	F_d	force due to interaction with fluid

A net force ALWAYS causes an acceleration, acceleration ALWAYS in the direction of the net force

Inertial reference frame: reference frame where there is no acceleration or rotation present

Inertia: It is the ability of an object to resist change in its state of motion

Equilibrium: State of balanced forces, no motion or motion with constant velocity

Kinematics:

Recognize when to use kinematics, usually when given time, velocity, initial velocity, or distance.

$V = V_o + at$

$\Delta x = V_o t + \frac{1}{2} at^2$

$V_f^2 = V_o^2 + 2a\Delta x$

Newton's Laws

Newton's First Law of Motion (N1L)

In an inertial reference frame, an object at rest stays at rest an object in motion continues its motion along a straight line with constant velocity unless a nonzero net force acts on it (unbalanced external force)

$\boldsymbol{\Sigma F = 0}$

Newton's Second Law of Motion (N2L)

In an inertial reference frame, net external force cases object to accelerate. Acceleration will occur in the direction of net external force, and its magnitude will be directly proportional to the magnitude of the net external force and inversely proportional to the mass of the object.

(Force goes up, acceleration goes up), (Mass goes up, acceleration goes down)

$\boldsymbol{\Sigma F = ma}$ $\boldsymbol{\Sigma F = F_{net}}$

Newton's Third Law of Motion (N3L)
In an inertial reference frame, if object A applies a force on object B, then object B applies the same magnitude force in opposite direction at the <u>same time</u> on object A.
(Not a reaction force which implies there is a time delay when in reality, there isn't)

Difference between N2L and N3L:
N2L = multiple forces on one object
N3L = 2 objects and 1 force

Terminal Velocity

Only true at terminal velocity, when $F_d = F_g$
$F_d = \frac{1}{2} D\rho AV^2$
F_d = Drag force (N)
D = Drag coefficient (constant)
ρ (Rho) = Density of fluid (constant)
A = cross sectional area (m^2)
V = velocity (m/s)

As the angle θ (angle with the ceiling or horizontal) decreases or approaches 0˚, the tension increase as the angle increases and approaches 90˚, the tension decreases. (Think about lifting a book with arm stretched out)

Friction

2 kinds of frictional force: Static and Kinetic frictional force
- Static Frictional Force (F_{fs})
- Kinetic Frictional Force (F_{fk})
- Max static frictional force (F_{fmax})

Static:	Kinetic:
Prevents object from moving	Slows down moving objects
Opposes applied force	Acts in the opposite direction of motion
Static is proportional to force applied	Kinetic force is not proportional to force applied

μ = Mu = Coefficient of friction between 2 surfaces (roughness between surfaces
μ = between 0 and 1
Static > Kinetic $\mu_s > \mu_k$ $F_{fs} = \mu_s F_N$ $F_{fk} = \mu_k F_N$ $F_{fmax} = \mu_s F_N$
F_{fmax} is used ONLY to check if the object will move, compare to applied force (make sure same direction)

Critical Kinetic and Critical Slip Angle

Condition for mass to move down an inclined plane at constant velocity in terms of μ_k and θ?

Inclined plane with friction
a = 0 $Fg_{||} - F_{fmax} = 0$

$a = g(\sin\theta - \mu_k\cos\theta)$
$g(\sin\theta - \mu_k\cos\theta) = 0$
$\sin\theta - \mu_k\cos\theta = 0$
$\mu_k\cos\theta = \sin\theta$
$\mu_k = \sin\theta / \cos\theta$
$\boldsymbol{\mu_k = \tan\theta_{ck}}$

$Fg_{||} = F_{fmax}$
$mg\cos\theta = \mu_s F_N$
$mg\cos\theta = \mu_s mg\sin\theta$
$\cos\theta = \mu_s\sin\theta$
$\mu_s = \sin\theta / \cos\theta$
$\boldsymbol{\mu_s = \tan\theta_{cs}}$

$\mu_k = \tan\theta_{ck}$ = critical kinetic angle, angle for mass to slide with constant velocity

$\mu_s = \tan\theta_{cs}$ = critical slip angle, angle when mass is just holding on, no movement

$\tan\theta_{cs} > \tan\theta_{ck}$
Since $\mu_s > \mu_k$ therefore $\theta_{cs} > \theta_{cs}$

TIPS ON HOW TO SOLVE PROBLEMS:

1. Draw free body diagram first
2. Draw pseudo free body diagram if necessary (when forces aren't only in X or Y direction)
3. Find the sum of forces in x and y
4. ALWAYS use angles in reference to the horizontal, try looking at alternate interior angles
5. If using angles from the horizontal
   a. X component = $\cos\theta$, Y component = $\sin\theta$
   b. X (parallel) component = $\sin\theta$, Y (perpendicular) component = $\cos\theta$, for inclined planes
   c. Verify/Check your trig values by "isolating" the triangle in your head and seeing if it makes sense, (is it really the opposite side of the angle?)
6. Only on frictionless incline planes: $a = g\sin\theta$, $F_N = mg\cos\theta$
7. If the system is at rest or constant velocity, all forces must balance out in x and y- direction
8. "Pulling" a heavier mass = more tension
9. Elevator problems: accelerating upward will have greater F_N up, accelerating downward will have greater F_g down, (free-body diagrams show acceleration's direction)
10. Tension is constant throughout the entirety of the rope, no magical rope
11. Try to equate tensions if possible, definitely equate tensions if there are 2 unknowns in your equations
12. It takes any amount of mass greater than 0 to move an object on a frictionless Atwoods machine (elephant and the mouse analogy)
    a. The greatest acceleration of the system is when Tension = 0 N and the smallest acceleration in the system is when Tension = m_2g, max tension is string is mg
13. Compare F_{fmax} to applied force in the same direction as the friction
14. When solving any problem/system with friction, find 4 things using columns (static, kinetic, and $\Sigma F = ma$):
    a. F_N (used to calculate frictional force)
    b. F_{fmax} (checks if the object will move, compare this to $F_{applied}$)
    c. F_{fk} (conditional, only if the object is moving)
    d. acceleration (conditional, only if the object is moving)

UNIT 4: Circular Motion and Gravitation

Fundamentals of Unit 4 Physics

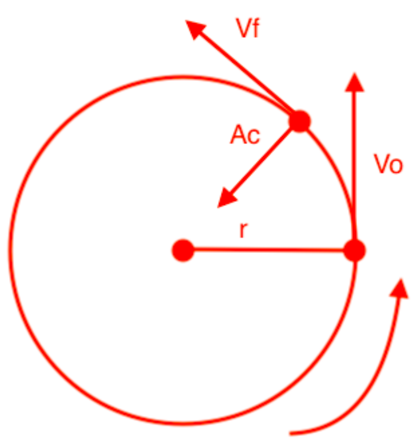
Circular motion is when objects move in a circle with constant speed
Major Concept: If an object is moving in a circle, it changes its direction at every point on the circumference of the circle, it accelerates toward the center of the circle
Changing velocity = change in magnitude OR change in direction

Misconception: Centripetal force is a new type of force
Truth: Any force (net force) that can make an object move in a circle serves as centripetal force

- Centripetal force (F_c) can be tension (T), normal force (F_N), gravity (F_g), static friction (F_{fs}) etc.

Sources of centripetal force:

1. Pail whirling in circle = tension in string
2. Water in pail whirling in a circle = normal force from bottom of pail
3. Earth moving in orbit around the sun = sun's gravitational force
4. Turning car on flat circular curve = static frictional force by the road
5. Bobsled turning on a banked circular curve (inclined plane) = horizontal component of normal force



a_c = centripetal acceleration
$a_c = \Delta v / t$
Δv = accelerating towards center
$\Delta v = v_f - v_o$

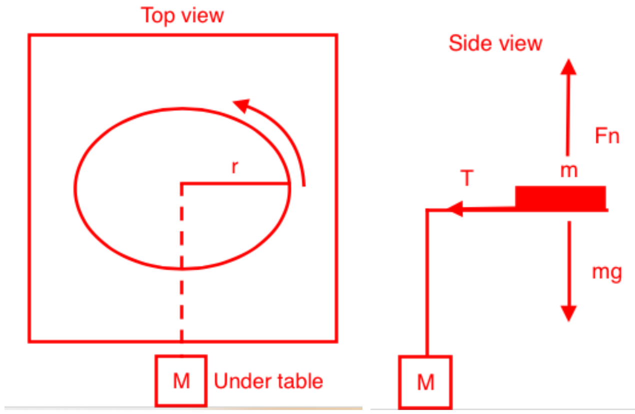
Newton's Second Law for centripetal acceleration:
$\mathbf{\Sigma F = ma_c}$
$\mathbf{\Sigma F = m(v^2 / r)}$

$a_c = v^2 / r$

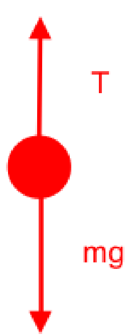
v = speed of object
r = radius of circle
m = mass of object moving in a circle

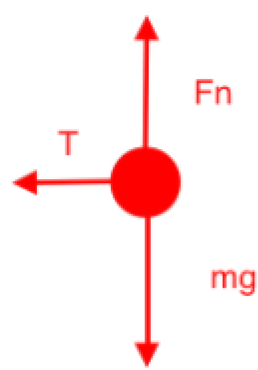
Case studies in a circular motion

Case 1: Horizontal circular motion on a flat frictionless table



M: hanging mass that provides tension for the puck and thereby centripetal force for the puck to move in a circle
m: mass of the puck
v = speed of puck
r = radius of circular path





$\Sigma F_y = 0$
$T - Mg = 0$
$T = Mg$

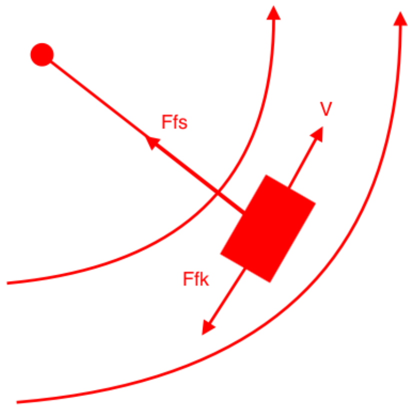
$\Sigma F_x = ma_c$
$T = ma_c$
$T = m\,(v^2 / r)$

$\Sigma F_y = 0$
$F_N - mg = 0$
$F_N = mg$

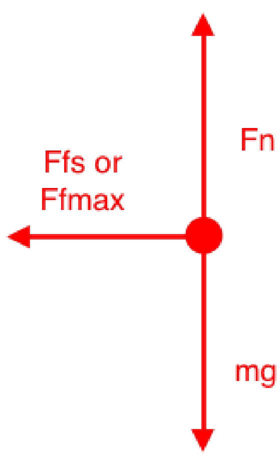
Equate Tensions
$Mg = m\,(v^2 / r)$

Double speed = Quadruples tension
Half the radius = Doubles tension

Case 2: Car on a flat circular curve



m = mass of the car
r = radius of curve
v = car's speed
Static frictional force provides centripetal force because radius of the curve isn't changing. Car doesn't move in direction of F_{fs}.
$0 \leq F_{fs} \leq F_{fmax}$



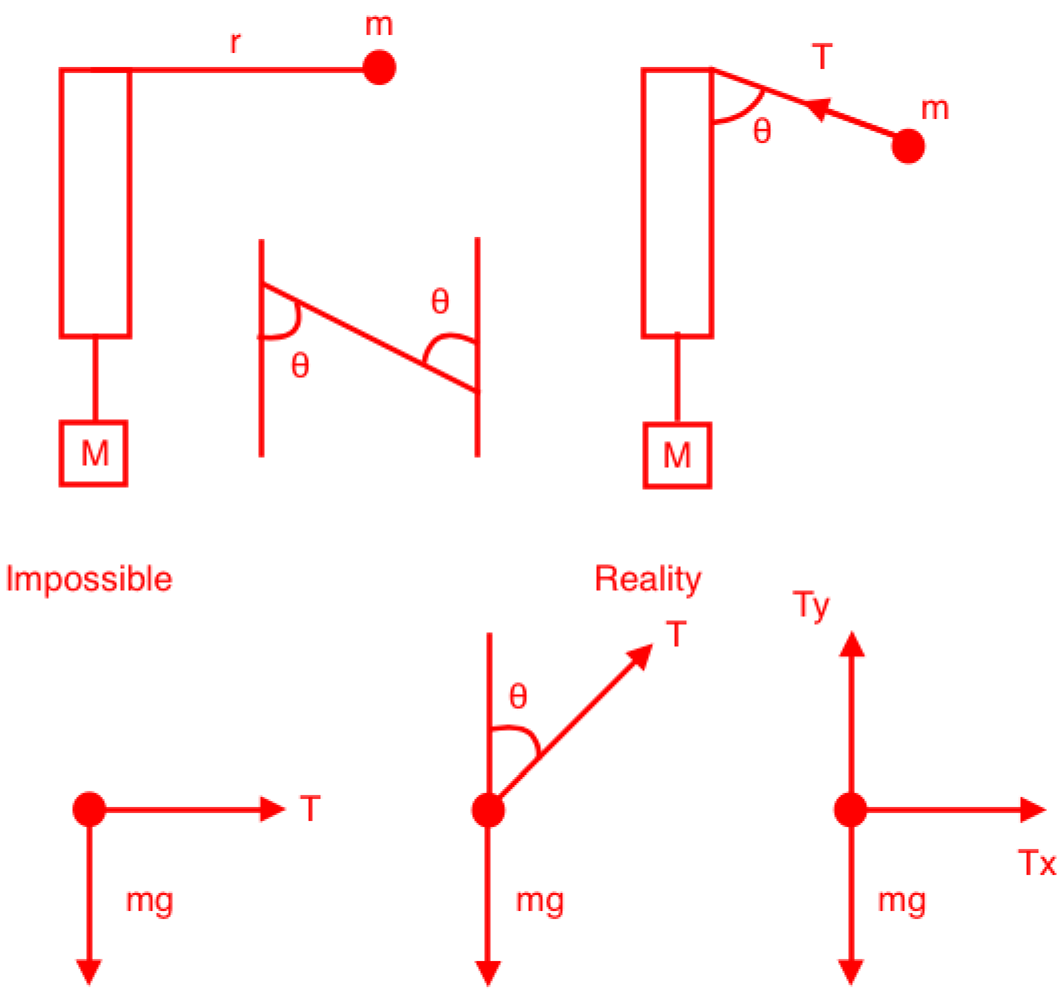
$\Sigma F_x = ma_c$	$\Sigma F_y = 0$
$F_{fs} = m\,(v^2 / r)$	$F_N - mg = 0$
$\mu_s F_N = m\,(v^2 / r)$	$F_N = mg$
$F_N = (mv^2 / r) / \mu_s$	

Equate normal force (F_N)
$mg = (mv^2 / r) / \mu_s$
$g = v^2 / \mu_s r$
$v^2 = \mu_s rg$
$v = \sqrt{\mu_s rg}$ = evaluating a safe speed

$v^2 / r = \mu_s g$

$a_c = \mu_s g$

Case 3: Whirlygig



Since angle (θ) is measured from the vertical,
the x component is calculated using sinθ
while the y component is calculated using cosθ

$T_x = T\sin\theta$ $\quad$ $T_y = T\cos\theta$

$\Sigma F_x = ma_c$

$T_x = m (v^2 / r)$

$T\sin\theta = m (v^2 / r)$

$\Sigma F_y = 0$

$T_y - mg = 0$

$T_y = mg$

$T\cos\theta = mg$

$T = mg / \cos\theta$

Equate/Substitute Tensions

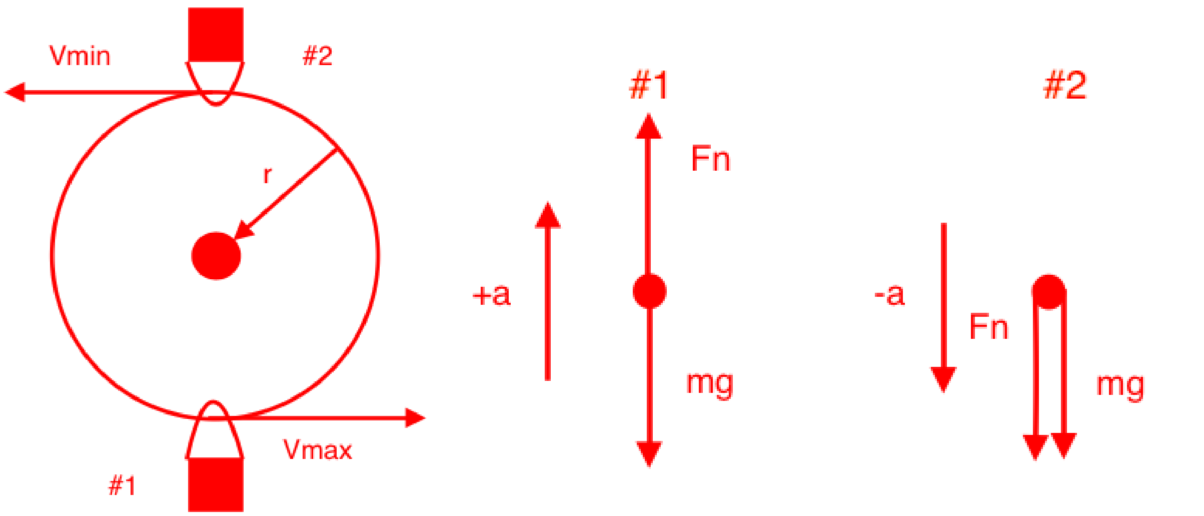
$(mg / \cos\theta) \sin\theta = mv^2 / r$

$g\sin\theta / \cos\theta = v^2 / r$

$g\tan\theta = v^2 / r$

$g\tan\theta = a_c$

Case 4: Whirling a pail of water in a circle



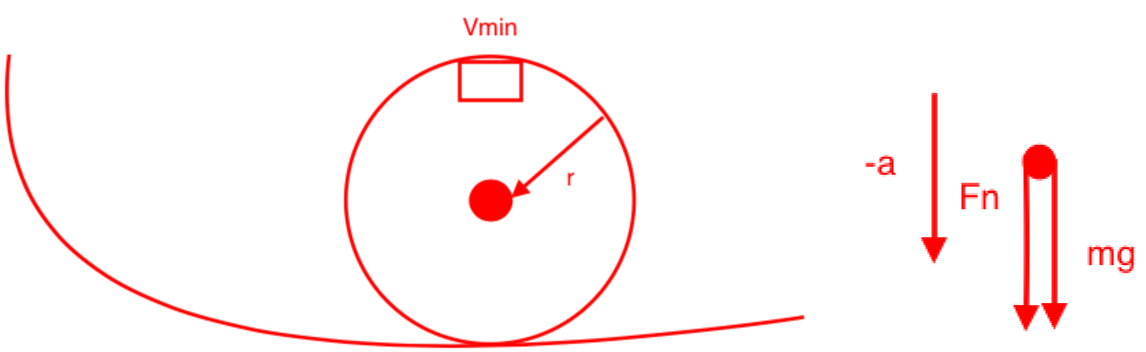
$\Sigma F_y = ma_c$
$F_N - mg = m (v^2 / r)$
$F_N = mv^2 / r + mg$

$F_N = m (v^2 / r + g)$
$F_N = m (a_c + g)$

$\Sigma F_y = ma_c$
$- F_N - mg = - m (v^2 / r)$
$F_N + mg = mv^2 / r$
$F_N = mv^2 / r - mg$
$F_N = m (v^2 / r - g)$
$F_N = m (a_c - g)$

Finding V_{min} when $F_N = 0$ at the top of the circle (#2)
$F_N = mv^2 / r - mg$
$0 = mv^2 / r - mg$
$mg = mv^2 / r$
$g = v^2 / r = a_c$
$v^2 = rg$
$v_{min} = \sqrt{rg}$

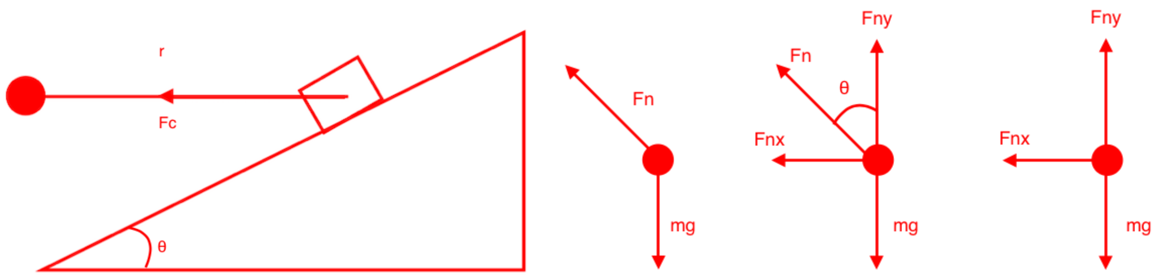
Case 5: Roller coaster with a vertical loop-the-loop and finding required minimum speed



What is V_{min} in terms of m, r, and fundamental constants?
$\Sigma F_y = ma_c$
$-F_N - mg = -mv^2 / r$
V_{min} will occur when $F_N = 0$
$-mg = -mv^2 / r$

$g = v^2 / r = a_c$
$v^2 = rg$
$v_{min} = \sqrt{rg}$

Case 6: Banked curves without friction



Since angle (θ) is measured from vertical:
$F_{Nx} = F_N \sin\theta$
$F_{Ny} = F_N \cos\theta$

Evaluate a proper / safe speed for the turns
Horizontal component of the normal force (F_N) provides the centripetal force

$\Sigma F_x = ma_c$	$\Sigma F_y = 0$
$F_{Nx} = mv^2 / r$	$F_{Ny} - mg = 0$
$F_N \sin\theta = mv^2 / r$	$F_N \cos\theta = mg$
	$F_N = mg / \cos\theta$

Equate / Substitute normal force (F_N)
$(mg / \cos\theta) \sin\theta = mv^2 / r$
$g\tan\theta = v^2 / r = a_c$

Proper banked angle for speed/radius
$g\tan\theta = v^2 / r$
$\tan\theta = v^2 / rg$
$v^2 = rg\tan\theta$
$v = \sqrt{rg\tan\theta}$ Proper/safe speed for banked angle, faster velocity = up the incline, vice versa
$r = v^2 / g\tan\theta$ Proper turn radius for speed v

Period of Revolution (T)

The time it takes for an object to go around the circle of radius r with constant speed once
Speed = distance / time = circumference / period
V = velocity
T = period in seconds
$V = 2\pi r / T$
$T = 2\pi r / V$

Tangential Velocity

The velocity of an object moving in a circle of radius r that is tangent to the circular path of an object
Tangential velocity is a function of radius. It varies linearly with the radius.

The tangential velocity increases with radius. The point on the disk farthest from the center has a higher tangential velocity. The point at the center has zero velocity.
Tangential velocity is tangent to the circumference of the circle and perpendicular to the centripetal acceleration.
v = tangential velocity (m/s)
r = radius (m)
ω = angular velocity (rad/s or RPM)
Tangent velocity = radius x angular velocity
V = r * ω
Velocity DISREGARDS mass
Velocity decreases as radius decreases

Angular / Rotational Velocity

How fast something is rotating defines its angular or rotational velocity
ω = angular velocity (rad/s or RPM)
Arc length (s) = radius x angle
s = r * Δθ
1 radian = when the arc length (s) is exactly equal to radius ® the object has turned an angle of one radian
2π radians = 1 revolution
Circumference = r (2π)
s / r = Δθ (1 radian)

Angular displacement
ω = Δθ / t

Dimensional Analysis and Converting Units

Converting Units by dimensional analysis
Placing numerator units in the denominator so that they cancel and vice versa

Converting RPM to rad/s: Multiply by 2π / 60
Converting rad/s to RPM: Multiply by 60 / 2π

$$\frac{5000\, revolutions}{minute} * \frac{1\, minute}{60\, seconds} * \frac{2\pi\, radians}{1\, revolution} = 523\, rad/s$$

Newton's Gravitation Law (NGL)

Universal Gravitation: Every mass in the universe attracts every other mass in the universe by a force which is proportional to the product of the two masses and inversely proportional to the square of the distance between them

$F_g \propto m_1 m_2$
$F_g \propto 1 / r^2$
$F_g \propto m_1 m_2 / r^2$
$\mathbf{F_g = G m_1 m_2 / r^2}$

Universal gravitational constant (G) = 6.67×10^{-11} N * m / kg^2

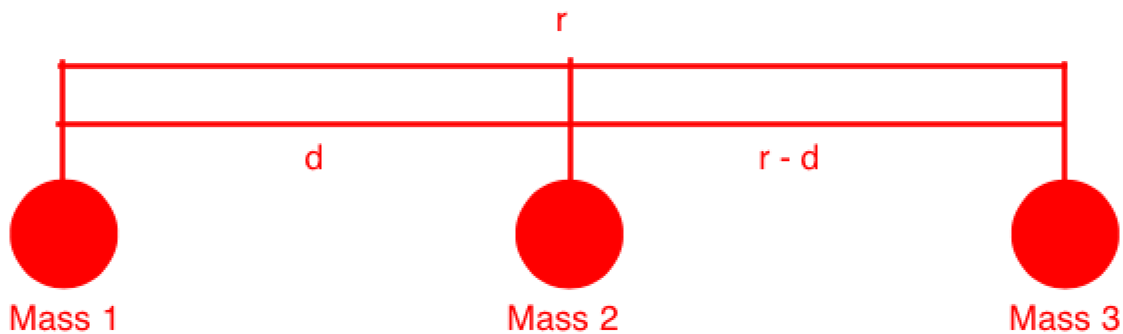
$M_E = 5.98 \times 10^{24}$ kg
$R_E = 6.38 \times 10^6$ m

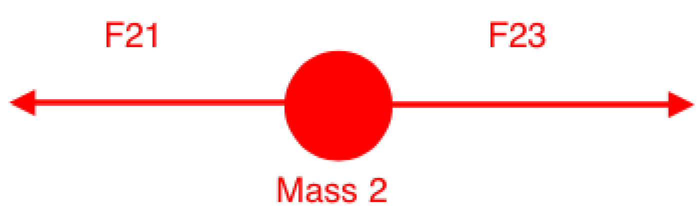
Force between person and Earth if mass = 80 kg
$F_g = G m_E m_{person} / r^2$
$F_g = (6.67 \times 10^{-11})(5.98 \times 10^{24})(80) / (6.38 \times 10^6)^2$
$F_g = 784$ N
$F_g = mg = 80(9.8) = 784$ N
$g = G m_E / (R_E)^2 = (6.67 \times 10^{-11})(5.98 \times 10^{24}) / (6.38 \times 10^6)^2 = 9.8$ m/s^2 = gravity for Earth

Proportional Reasoning

$F_g = G \_\_m_1 \_\_m_2 / (\_\_r)^2 = F_g$
$F_g = G\ 2m_1\ 2m_2 / (3r)^2 = 2 * 2 / 3^2 = 4/9 * F_g$
$F_g = G\ 4m_1\ 1m_2 / (0.5r)^2 = 4 * 1 / 0.5^2 = 4/0.25 = 8 * F_g$

Three-Body Problems





F_{21} = force of gravity on mass 2 due to mass 1
F_{23} = force of gravity on mass 2 due to mass 3

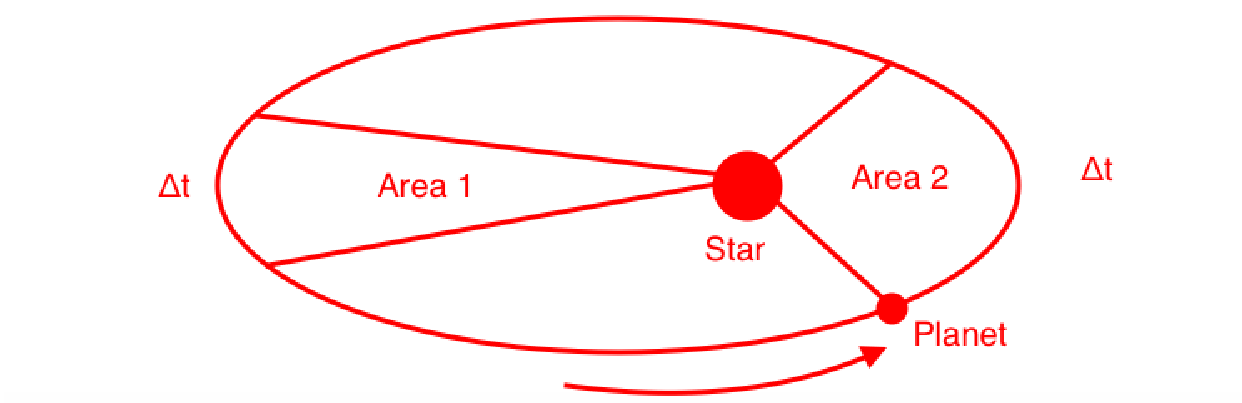
Find acceleration
$\Sigma F_x = ma$
$F_{23} - F_{21} = m_2a$
$Gm_2m_3 / (r-d)^2 - Gm_2m_1 / r^2 = m_2a$

If there is no acceleration or unbalanced net force
$\Sigma F_x = 0$
$F_{23} - F_{21} = 0$
$F_{23} = F_{21}$
$Gm_2m_3 / (r-d)^2 = Gm_2m_1 / r^2$
$Gm_2m_3 * r^2 = Gm_2m_1 * (r-d)^2$
$m_3r^2 = m_1(r-d)^2$

Kepler's Laws of Planetary Motion (K1L, K2L, K3L)

Kepler's First Law (K1L)
Orbit of a planet around its star is elliptical with star located at one of the foci (plural for focus)

Kepler's Second Law (K2L)
An imaginary line drawn from the star to the planet in the orbital plane, sweeps equal area in space in equal time intervals. If Δt are equal, then Area 1 will be equal to Area 2.



Planet's velocity slows down as planet gets farther because gravitation force is weaker

Kepler's Third Law (K3L)
The square of the orbital period of a planet is directly proportional to the cube of the semi major ais of the planet's orbit around the star.

T = orbital period
a = semi major axis
$T^2 \propto a^3$

Assuming circular orbit in AP Physics 1.

T = orbital period
r = radius of the orbit
$T^2 \propto r^3$

Kepler's Constant and Deriving equations of velocity, radius, and period (T) from Kepler's Constant

$F_g = F_c$
$Gm_1m_2 / r^2 = m_2v^2 / r$
$Gm_1m_2r = m_2v^2r^2$
$Gm_1 = v^2r$ m_1 = body of mass that is being orbited (the center)
$v^2 = Gm_1 / r$

$$v = \sqrt{\frac{Gm_1}{r}}$$

As radius increases, velocity decreases and vice versa

$v^2 = Gm_1 / r$
$v = 2\pi r / T$
$(2\pi r / T)^2 = Gm_1 / r$
$4\pi^2r^2 / T^2 = Gm_1 / r$

$Gm_1T^2 = 4\pi^2r^3$ K3L = $T^2 \propto r^3$
$T^2 = (4\pi^2 / Gm_1)\, r^3$

$(4\pi^2 / Gm_1)$ = Kepler's Constant for masses orbiting around m_1 or the sun

$$r = \sqrt[3]{\frac{T^2Gm_1}{4\pi^2}}$$

Period goes up, radius goes up

<u>TIPS ON HOW TO SOLVE PROBLEMS:</u>

1. ALL USEFUL FORMULAS:
   a. $F_g = F_c$
   b. $v = r * \omega$
   c. $v = 2\pi r/T$ $T = 2\pi r/v$
   d. $F_g = Gm_1m_2 / r^2$
   e. $a_c = g = v^2 / r$

      K3L:
   f. $T^2GM_S = 4\pi^2r^3$ $T^2 = (4\pi^2 / GM_s)\, r^3$

g. $v = \sqrt{\dfrac{Gm_1}{r}}$ $r = \sqrt[3]{\dfrac{T^2 Gm_1}{4\pi^2}}$

2. Start off with $F_g = F_c$ for gravitational problems in space
3. Multiply coefficients when doing proportional reasoning
4. Period (T) increases, radius (r) increases, altitude (h) increases, velocity (v) decreases
5. Minimum velocity occurs when there is no normal force ($F_N = 0$)
    a. Usually at the top of a vertical circle (loop-the-loop or whirling pail of water)
6. Acceleration for something moving in a circle is ALWAYS towards center of circle
    a. Acceleration is perpendicular inwards towards the center to tangential velocity
7. When angle (θ) is measured from vertical, the horizontal component is obtained with $\sin\theta$, vertical component is obtained with $\cos\theta$
8. When on a carousel or merry-go-round, use $v = r * \omega$ because the angular velocity is the same throughout the entire rotating disk, that means bigger radius means bigger velocity
9. When in space such as orbiting celestial objects (star, moon, Earth), use $T^2GM_S = 4\pi^2r^3$ because angular velocity is not the same for all objects orbiting a mass, that means bigger radius means smaller velocity according to the K3L equation for velocity

UNIT 5: Work, Power, and Energy

Fundamentals of Unit 5 Physics

Work (J) : Transfer of energy to or from a system by the action of a force

- "Applying a force, and a displacement has occurred, the work is done on that object or that system."
- "1 Joule of work is done when 1 Newton of force is applied to a system or an object and 1 meter of Δx has occurred"
- **$W = | F | | \Delta x | \cos\theta$**
    - W = work [Joules J]
    - F = force [Newtons N]
    - Δx = displacement [meters m]
    - θ = angle between the force vector and displacement vector [degrees ˚]
- Net-work = sum of all work done on an object
- Positive work = system gained kinetic energy (equal to work) = increase in velocity or speeding up
- Negative work = system loss kinetic energy (equal to work) = decrease in velocity or slowing down
    - Ball going up then down has negative work going up then positive work going down

Kinetic Energy (J) : Energy of motion of an object of mass m

- Scalar unit and has units joules
- **$KE = ½ mv^2$**
- ΔKE = change in kinetic energy = W = work done on system
    - $W = \Delta KE$ [Kinetic energy - Work Theorem]
    - $\Delta KE = KE_f - KE_o = ½ mV_f^2 - ½ mV_o^2$

Potential Energy (J) : (Stored energy)

- Energy due to the position of an object relative to a <u>reference point</u>
- Equal in magnitude to the work done by <u>an external force</u>
- **Kinds of Potential Energy:**
    - Gravitational Potential Energy
        - $U_g = mg\Delta y = mgh$
        - Δy = height, distance from where $U_g = 0$
        - In gravitational field
        - Energy due to height/depth/vertical position near Earth's surface
        - Equal to the <u>minimum</u> work done by an external force AGAINST gravity
    - Elastic or Spring Potential Energy
        - $U_s = ½ k\Delta x^2$
        - F_s [N] = spring force
        - Δx [m] = displacement from spring's equilibrium position
        - k [N/m] = spring constant, stiffness of spring
        - $F_s \propto -\Delta x$

- $F_s = -k\Delta x$
  - F_s always opposes Δx direction

Mechanical Energy (J) : Sum of all kinetic energies and all potential energies in a system
- $E_{mech} = U_g + U_s + KE$
- $E_{mech} = U$ (potential) + K (kinetic)

Conservation of Mechanical Energy : Mechanical Energy of a system stays constant in the absence of resistive forces (friction or drag)

- Need these three lines for all mechanical energy problems (Mech energy conserved) :
  - $\Delta E_{mech} = 0$
  - $E_{mechi} = E_{mechf}$
  - $U_{go} + U_{so} + K_o = U_{gf} + U_{sf} + K_f$

- Don't have to include work in mechanical energy equation
- Sum of all potential and kinetic energies is the same regardless of where on that system we measure that sum

Mechanical Energy is NOT conserved : Loss of mechanical energy translates to heat generated within the system. (Work done by resistive force)

- Need these three lines for all mechanical energy problems with resistive forces (Mech energy not conserved) :
- $\Delta E_{mech} = 0$
- $E_{mechi} = E_{mechf} + | W_{resistive} |$
- $U_{go} + U_{so} + K_o = U_{gf} + U_{sf} + K_f + W_{resistive}$

- $E_{mechi} > E_{mechf}$
- $W_{resistive}$ is always negative, so there is an absolute value sign

Power (W) : Rate of doing work, or rate of energy dissipation, consumption, generation, or transformation
- $P = W / t$
  - P = power [watts w]
  - W = work [joules J]
  - t = time [seconds s]
- 1 Horsepower [hp] = 746 Watts
- 1 kilowatt [kW] = 1000 Watts
- $P = \frac{W}{t} = \frac{F\Delta x}{t} = \frac{ma\Delta x}{t} = mav = \frac{\Delta KE}{t}$

Kinetic energy - Work Theorem

$W = \Delta KE$ [Kinetic energy - Work Theorem]

$\Delta KE = KE_f - KE_o = ½\, mV_f^2 - ½\, mV_o^2$

- Can be used to find velocities of systems when given work or numbers to be able to calculate work

Work for Objects in Orbit/Forces which are Perpendicular to the Displacement Vectors

Work done by perpendicular forces are **0 Joules.**

Angle between force vector and displacement/velocity vector is 90˚

cos(90˚) = 0, therefore, W = FΔxcos(90˚) will always be 0 Joules

Objects in orbit have force vectors as centripetal force, pointing towards the center circle, perpendicular to the tangent velocity vector. Therefore, objects in orbit experience no work by gravity.

Work for Forces in Different Directions (Positive, Negative, and No Work)

Positive Work = Any force vector in the same direction of displacement vector until 90˚

No Work = Any force vector perpendicular to or at 90˚ from displacement vector

Negative Work = Any force vector in the opposite direction of displacement vector until 90˚

Work Done by Resistive Forces

Resistive forces are antiparallel to displacement vectors, resulting in negative work done.

Resistive forces are Friction and Drag

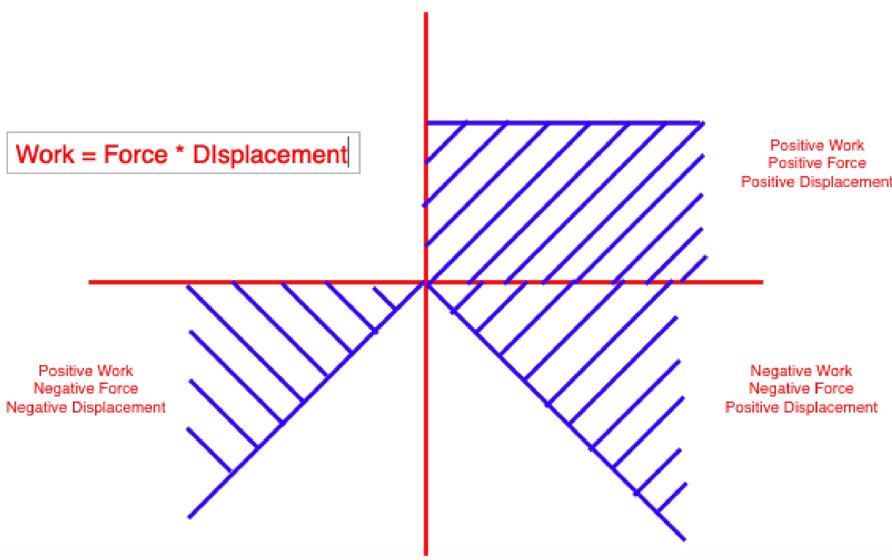
Calculating Work from Graphs

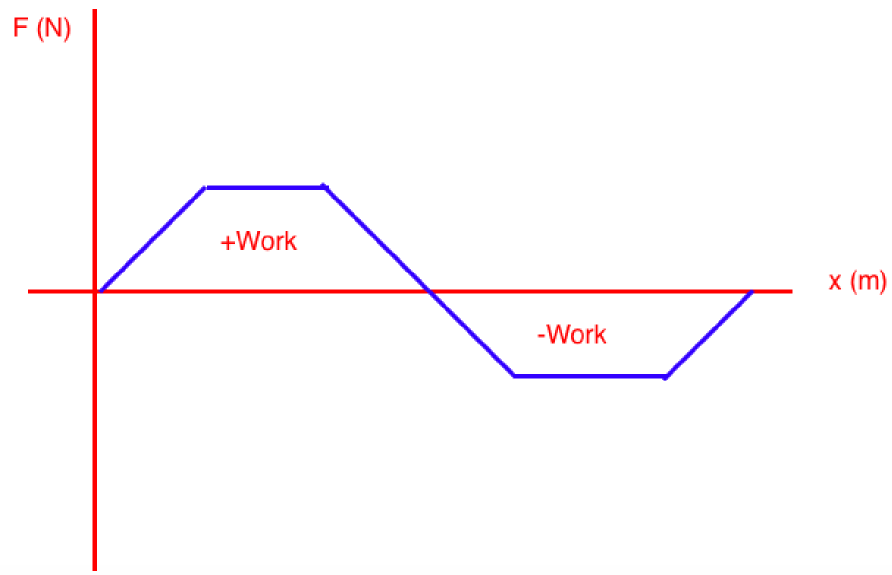
Area under the curve on a force (N) vs displacement (Δx) graph

Area under the curve results in Net-Work done on the system

Quadrants 1 and 3 are positive work

Quadrants 2 and 4 are negative work





TIPS ON HOW TO SOLVE PROBLEMS:

1. **ALL IMPORTANT EQUATIONS:**
   a. $W = |F||\Delta x|\cos\theta$
   b. $\Delta KE = KE_f - KE_o = \frac{1}{2} mV_f^2 - \frac{1}{2} mV_o^2$
   c. $W = \Delta KE$
   d. $E_{mechi} = E_{mechf}$ (No resistive forces)
      i. $U_{go} + U_{so} + K_o = U_{gf} + U_{sf} + K_f$
         1. $U_g = mg\Delta y$
         2. $U_s = \frac{1}{2} k\Delta x^2$
         3. $K = \frac{1}{2} mV^2$
   e. $E_{mechi} = E_{mechf} + |W_{resistive}|$ (Resistive forces)
      i. $U_{go} + U_{so} + K_o = U_{gf} + U_{sf} + K_f + W_{resistive}$
2. Write down all known variables and use them to determine which equation to use
3. When do mechanical energy questions, figure out which terms are 0
   a. Change terms such as U_g and K to their equations and figure out if the Δy or V is 0 at that position
4. **Might be useful to change mechanical energy equation**
   a. $U_{go} + K_o = U_{gf} + K_f$
   b. $U_{go} = U_{gf} + K_f - K_o$
   c. $U_{go} = U_{gf} + \Delta KE$
5. Set $U_g = 0$ to the ground or the bottom of the system, make sure all Δy is measured from $U_g = 0$
6. For power problems, derive an equation from $P = W / t$ that is able to utilize all given variables
7. When dropping an object from a height, $U_{go} = K_f$
8. When launching an object from the ground, $K_o = U_{gf}$
9. A spring that is farther from its equilibrium position has more spring potential energy, more Δx
10. For power problems, if given velocity and time, don't try to calculate acceleration, just use $P = \Delta KE/t$ rather than $P = mav$

UNIT 6: Impulse, Momentum, and Center of Mass

Fundamentals of Unit 6 Physics

Momentum : Numerical degree or difficulty of how difficult or how easy it is to stop a moving object

- $p = mv$ [kg*m / s]   instantaneous momentum
    - Direction of momentum is determined by velocity
- $\Delta p = m\Delta v$ [kg*m / s]   change in momentum
    - Direction of change in momentum is determined by change in velocity
    - $\Delta v = v_f - v_o$

Impulse : equal to change in momentum (Δp)

- $J = \Delta p$
- $J = m\Delta v$ [kg*m / s]
- $J = F * t$ [N * s]
- Impulse and Force (N3L) would be the same in magnitude in a collision between two objects, the only difference is that impulse and force are in **opposite directions**
- Impulse can be calculated as area under a force vs time graph (units match up)

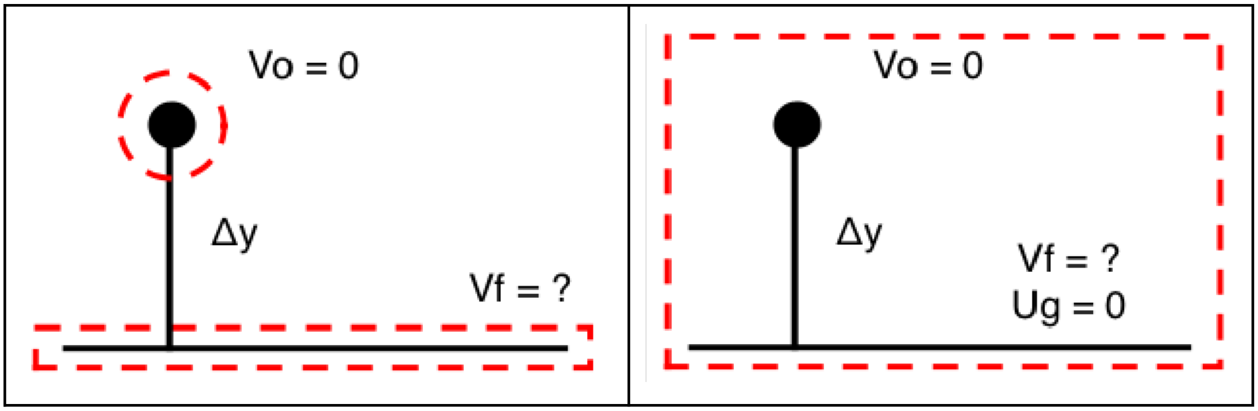
Impulse - Momentum Theorem

- Impulse = F * t
- Change in momentum = $m\Delta v$
- Impulse = Change in momentum
- $F * t = m\Delta v$
- Change in momentum is related to NET EXTERNAL FORCE
- Direction of impulse is the same as Force (F), change in velocity (Δv), acceleration (a)

Open vs Closed Systems

Open Systems	Closed Systems

- External forces are allowed to enter or exit system boundaries
- Must apply theorems:
    - W - KE Theorem
    - I - M Theorem
	- External forces are NOT allowed to enter or exit system boundaries
	- Must apply conservation laws:
	    - Energy Conservation
	    - Momentum Conservation



Conservation of Momentum (Closed Systems)

- Combining multiple open systems into one single closed system
- Interactions between open systems now become internal interactions
    - Internal interactions within the closed system DO NOT change the momentum of the closed system
- Only possible without external forces
- The total momentum of the system initially is equal to the total momentum of the system at final

$$\Delta p = 0 \qquad \Delta\Sigma F_{external} = 0$$

$$P_o = P_f$$

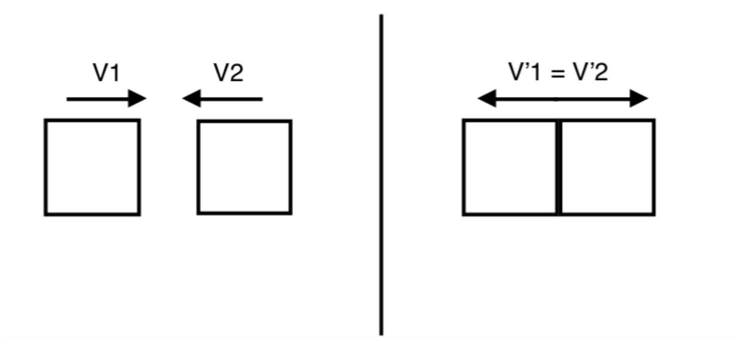
$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

Collision Types: (Closed Systems)

1. Inelastic Collision : particles can stick together or they can bounce off of each other. Momentum is conserved and kinetic energy decreases.

   $$\Delta p = 0 \qquad \Delta\Sigma F_{external} = 0 \qquad \Delta KE < 0 \qquad \Delta E_{mech} = 0$$

   Examples = Trains colliding and interlocking, cars crashing and interlocking, players tackling and interlocking

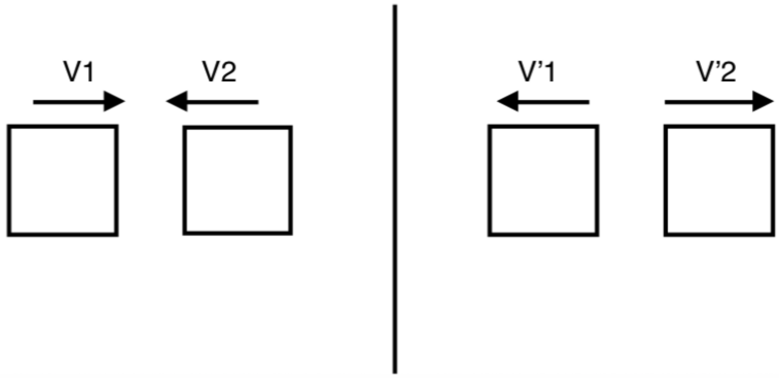


- Final velocities for both masses are the same: $V'_1 = V'_2 = V'$
    - $m_1v_1 + m_2v_2 = (m_1 + m_2)v'$

2. Elastic Collision : particles bounce off of each other. Momentum and kinetic energy is conserved.

$\Delta p = 0$ $\Delta\Sigma F_{external} = 0$ $\Delta KE = 0$ $\Delta E_{mech} = 0$

Examples = Ball bouncing off wall

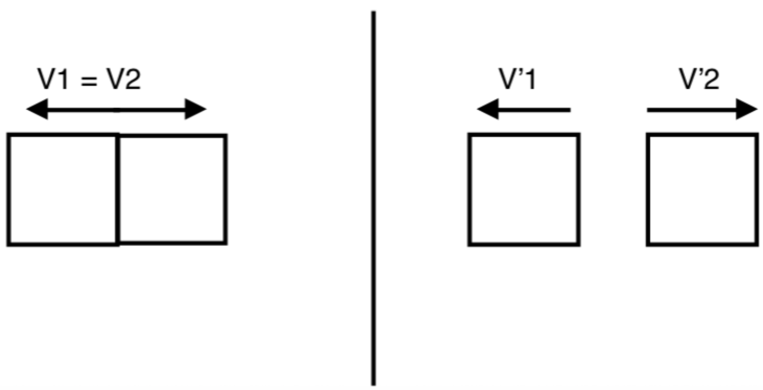


- Can use $\Delta P = 0$ and $\Delta KE = 0$ to get two different equations to solve a system of equations
  - Usually done when V'_1 and V^2_2 are unknown

3. Superelastic/Explosion Collision : One particle of the system breaks apart into many without outside forces entering the system. Momentum is conserved and kinetic energy increases.

$\Delta p = 0$ $\Delta\Sigma F_{external} = 0$ $\Delta KE > 0$ $\Delta E_{mech} = 0$

Examples = Rockets splitting apart into two stages



- Impulse / Change in Momentum is the same in magnitude
  - Impulse in equal and opposite directions
- The only thing different could be masses which would change ΔV
  - SMALL MASS = LARGE ΔV = FASTER
  - LARGE MASS = SMALL ΔV = SLOWER
- Explosion Equation: $(m_1 + m_2)v = m_1v'_1 + m_2v'_2$

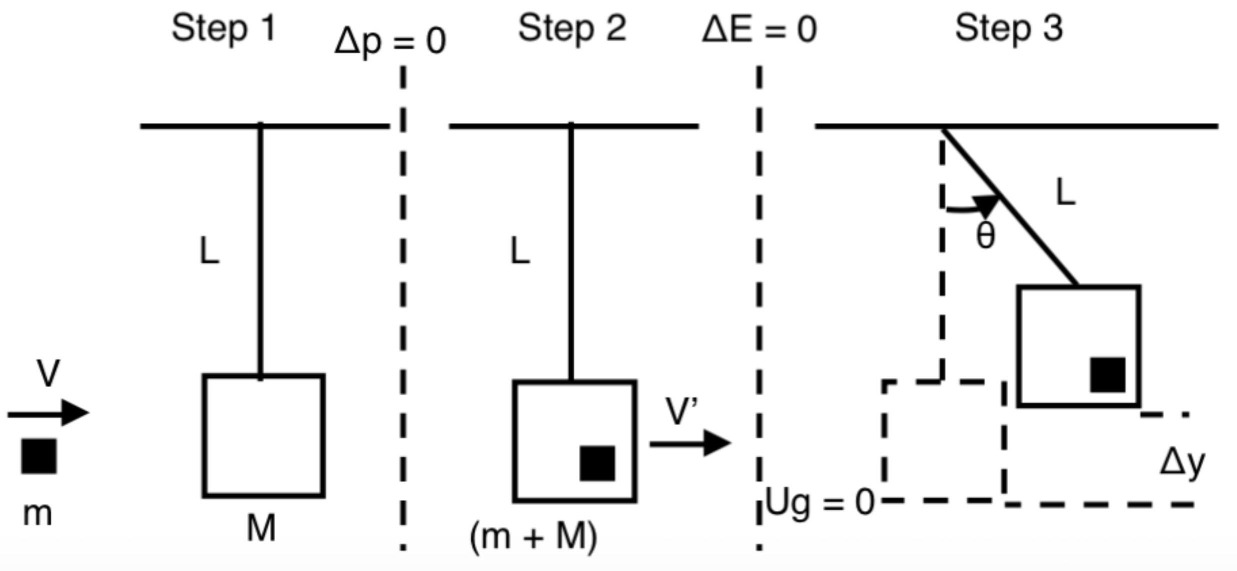
Problem Solving Steps

1. Identify the system or systems
2. Draw a diagram labeling all the given info and unknowns
3. Apply the appropriate physics principle based upon what is the unknown quantity

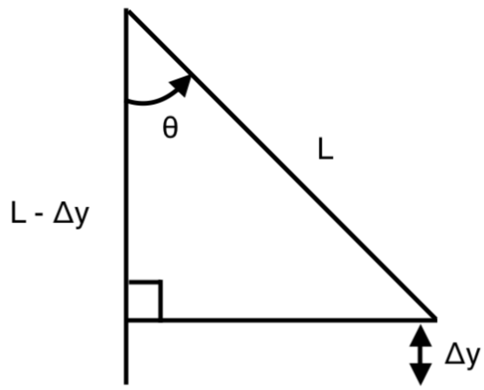
Ballistics Pendulum (Applying Conservation of Momentum and Conservation of Energy)

- **Assumptions :**
  - F_{drag} and F_g ignored
  - Completely inelastic collision, conservation of momentum applies
  - Kinetic Energy of the bullet-block system immediately after the collision will transform into gravitational potential energy when the pendulum swings to the highest point

- **Mass * velocity of the bullet = total momentum of the system**



- Steps 1-2, apply conservation of momentum (STAYS CONSTANT FOR THE TEST)
  - $\Delta p = 0$ $\Delta\Sigma F_{external} = 0$
  - $P_o = P_f$
  - $mv_1 + Mv_2 = (m + M)v'$
  - $mv = (m + M)v'$
  - $v = ((m + M) / m)v'$
  - $v = ((m + M) / m)\sqrt{2gL(1 - cos\theta)}$
- Steps 2-3, apply conservation of mechanical energy (METHOD TO GET V' CHANGES)
  - $\Delta E_{mech} = 0$
  - $K_o = U_{gf}$
  - $\frac{1}{2} mV'^2 = mg\Delta y$
  - $\frac{1}{2} mV'^2 = mg(L - Lcos\theta)$
  - $\frac{1}{2} V'^2 = g(L - Lcos\theta)$
  - **$V' = \sqrt{2gL(1 - cos\theta)}$**



Using triangle to get Δy

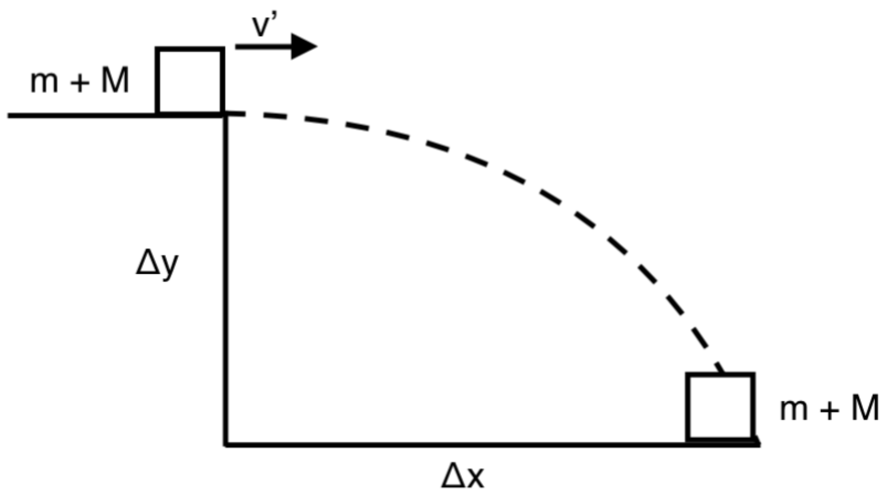
$\cos\theta = (L - \Delta y) / L$

$L\cos\theta = (L - \Delta y)$

$\Delta y = (L - L\cos\theta) = L(1 - \cos\theta)$

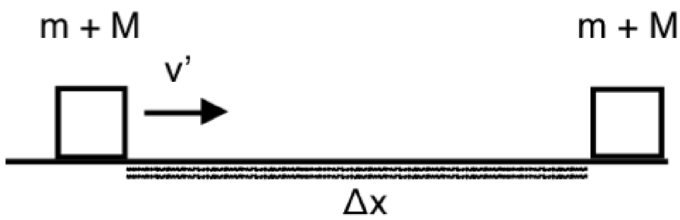
<u>THE DIFFERENT WAYS TO GET V’ (SPEED OF THE BLOCK AFTER COLLISION) :</u>

Projectile (Kinematics) :



Uniform Motion (x)	Accelerated Motion (y)
$V = V'$	$V_o = 0$
$\Delta x = \Delta x$	$V_f = ?$
$t = t$	$a = -10\ m/s^2$
	$\Delta y = \Delta y$
	$t = t$
$\Delta x = V' * t$	$\Delta x = V_o t + \frac{1}{2} at^2$
$\Delta x = V' * \sqrt{\frac{\Delta y}{-5}}$	$\Delta y = 0 + \frac{1}{2}(-10)t^2$
$\mathbf{V' = \frac{\Delta x}{\sqrt{\frac{\Delta y}{-5}}}}$	$\Delta y = -5t^2$
	$t = \sqrt{\frac{\Delta y}{-5}}$

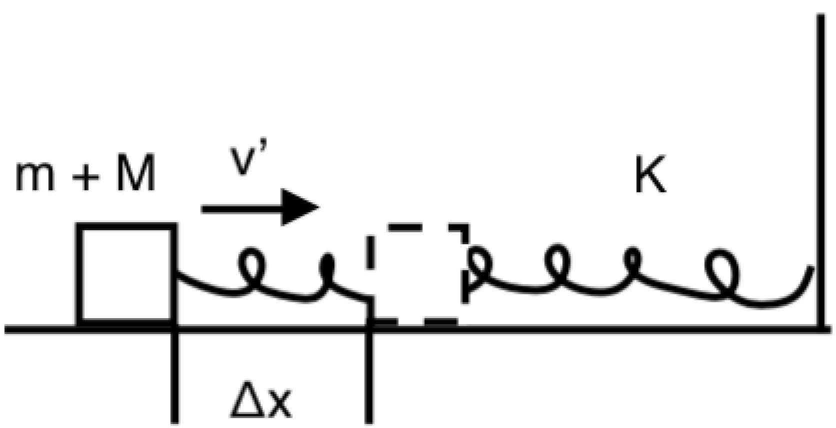
Block being stopped by Work done by friction ($\Delta E_{mech} = 0$) :



$\Delta E_{mech} = 0$
$K_o = W_{Friction}$
$\frac{1}{2}(m + M)V'^2 = F_{fk}\Delta x$
$\frac{1}{2}(m + M)V'^2 = \mu_k F_N \Delta x$
$\frac{1}{2}(m + M)V'^2 = \mu_k(m + M)g\Delta x$
$\frac{1}{2}V'^2 = \mu_k g\Delta x$

$$\mathbf{V'} = \sqrt{2\mu_k g\Delta x}$$

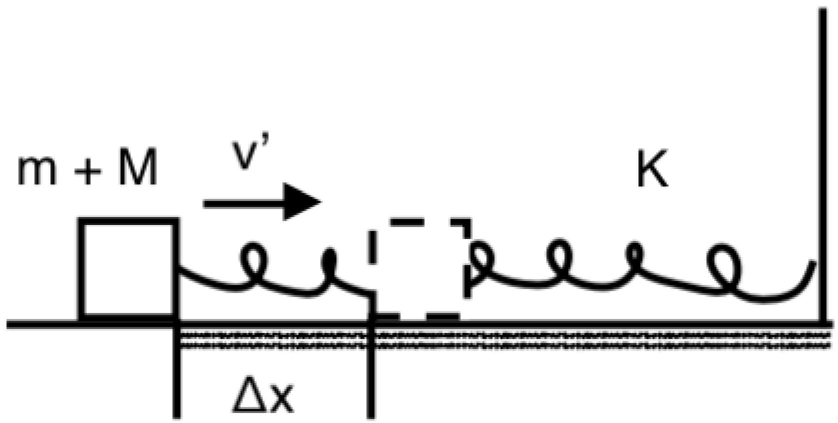
Block attached to spring that compresses ($\Delta E_{mech} = 0$) :



$\Delta E_{mech} = 0$
$K_o = U_{sf}$
$\frac{1}{2}(m + M)V'^2 = \frac{1}{2}k\Delta x^2$
$(m + M)V'^2 = k\Delta x^2$

$$\mathbf{V'} = \sqrt{\frac{k\Delta x^2}{m + M}}$$

Block attached to spring on friction surface (ΔE_{mech} = 0) :



$\Delta E_{mech} = 0$
$K_o = U_{sf} + W_{Friction}$
$\frac{1}{2}(m + M)V'^2 = \frac{1}{2}k\Delta x^2 + F_{fk}\Delta x$
$\frac{1}{2}(m + M)V'^2 = \frac{1}{2}k\Delta x^2 + \mu_k F_N \Delta x$
$\frac{1}{2}(m + M)V'^2 = \frac{1}{2}k\Delta x^2 + \mu_k(m + M)g\Delta x$
$\frac{1}{2}(m + M)V'^2 = \frac{1}{2}k\Delta x^2 + \mu_k(m + M)g\Delta x$

$$V' = \sqrt{\frac{\frac{1}{2}k\Delta x^2 + \mu_k(m+M)g\Delta x}{\frac{1}{2}(m+M)}}$$

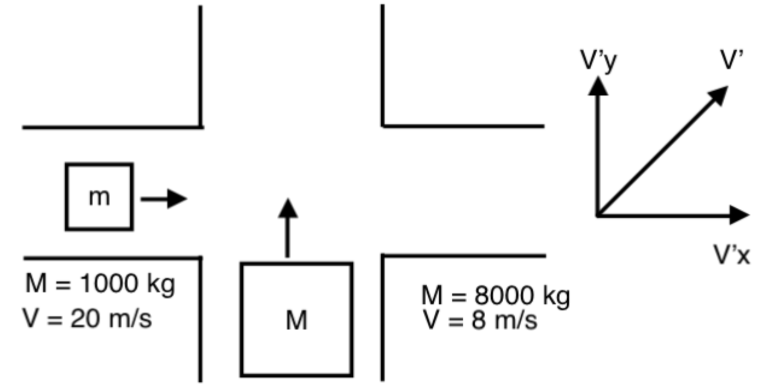
$$\mathbf{V'} = \sqrt{\frac{k\Delta x^2 + 2\mu_k(m+M)g\Delta x}{(m+M)}}$$

2-D Momentum and Impulse

Inelastic collision : $\Delta p = 0$, $\Delta k < 0$
$P_o = P_f$
Conservation of momentum: Final momentum same as initial momentum
CAN'T USE $(m_1 + m_2)v' = P_{total}$ to find P_{total}

This equation is used to find v'

This equation doesn't work for P_{total} because inelastic collision = loss of kinetic energy



X: Car $P_{car} = mV_{car} = 20{,}000$ kg*m / s
Y: Truck $P_{truck} = mV_{truck} = 64{,}000$ kg*m / s

$\Delta p_x = 0$
$m_1v_1 = (m_1 + m_2)V'_x$

$\Delta p_y = 0$
$m_2v_2 = (m_1 + m_2)V'_y$

$P_{total} = P_o = P_f$
$P_{total} = P_{car} + P_{truck}$
$P_{total} = P_x + P_y$ (Adding momentum vectors)
$P_{total} = \sqrt{P_x^2 + P_y^2}$
$P_{total} = \sqrt{(m_1v_1)^2 + (m_2v_2)^2}$
$P_{total} = \sqrt{(m_1 + m_2)^2 V_x^{2'} + (m_1 + m_2)^2 V_y^{2'}}$

After finding the momentum of the system, we can now find velocity after the collision (v')
$P_{total} = (m_1 + m_2)v'$
$v' = P_{total} / (m_1 + m_2)$

Center of Mass

A point in space where the mass of the entire system can be balanced.
For a rigid system (box, ball, etc.) the center of mass is the same as center of gravity.
Center of mass follows laws of physics, and can feel force, acceleration, impulse, change in velocity.

A system of multiple masses, the center of mass is a weighted location in space.
Center of mass on a 2D plane :
- Isolate x_{cm} and y_{cm} (calculate it independently)

Center of mass equation :
- $(m_1 + m_2)x_{cm} = m_1x_1 + m_2x_2$

- $x_{cm} = m_1x_1 + m_2x_2 / m_1 + m_2$

Exploding Projectiles (Rocket/Missile)

All explosions occur at apex
All initial momentum is transferred into the warhead
Center of mass follows the rocket as if it never exploded

Body of the rocket's final velocity = 0 ($V_1' = 0$)
Body of the rocket falls straight down to Range / 2 or X_{cm} / 2

Mass ratio = total rocket : warhead
Mass ratio of rocket to warhead is 200 : 1 so mass ratio of total rocket : warhead is 201 : 1
Solving exploding projectile questions
$\Delta p = 0$ $\Delta\Sigma F_{external} = 0$
$P_o = P_f$
$(m_1 + m_2)V_X = m_1v_1' + m_1v_2'$
$(m_1 + m_2)V_X = m_1v_2'$
$((m_1 + m_2) / m_1)V_X = v_2'$ $((m_1 + m_2) / m_1)$ = mass ratio of total rocket : warhead

- Bigger mass ratio = bigger velocity of warhead
- Smaller mass of warhead = bigger velocity of warhead
  - If the mass of the warhead is 10 times less than the mass of the total rocket (warhead and body), then the velocity of the warhead will be ten times more.

2 Methods to solve exploding projectile questions:

Method 1 (More intuitive and logical but requires more arithmetic and has greater room for error) More math, less concept	Method 2 (Personally think is easier because easy to get range and use X_{CM} formula because less numbers to deal with, also removes possibility of forgetting to add launch to R/2 distance) More concept, less math
1. Find initial velocities in x and y direction 2. Find time to reach apex 3. Find final velocity after collision 4. Find displacement from range/2 to landing 5. Add this displacement to the displacement from launch to r/2 (apex)	1. Find initial velocities in x and y direction 2. Find hangtime (total time in air) 3. Find range of the rocket (range = X_{cm}) 4. Find range / 2 5. Use X_{cm} and $X_{R/2}$ or range and range / 2 to get X_2 or landing position of warhead a. $(m_1+m_2)x_{CM} = m_1x_1+m_2x_2$ b. $(m_1+m_2)x_R = m_1x_{R/2}+m_2x_{warhead}$

TIPS ON HOW TO SOLVE PROBLEMS:

- Impulse will always have equal and opposite forces
- If two objects collide and they stick, it's inelastic and you can use inelastic equation immediately

- If two objects collide and the bounce off, its elastic and just use normal conservation of momentum
- If two objects start off together and they move away, it’s an explosion and you can use explosion equation immediately
- Momentum change or impulse will always be the same for the two objects colliding or exploding, so lower mass will mean higher speed and vice versa

UNIT 7: Oscillations and Simple Harmonic Motion

Fundamentals of Unit 7 Physics

Oscillations : Type of motion where an object moves back and forth about a certain point in space. This particular point in space is called the equilibrium point.

Simple Harmonic Motion : Type of oscillatory motion when 3 conditions are met:
1. Frequency and period of oscillation doesn't change
2. Presence of restoring force that causes object to exhibit SHM about an equilibrium point
3. Restoring force directly proportional to displacement from equilibrium

- Examples:
  - Mass-Spring system
  - Pendulum only when $\theta < 10^\circ$ (small angle approximation)

Period (s) : time for a complete back and forth motion or oscillation [seconds / 1 cycle]
- $T = 1 / f$
- $T_{mass\text{-}spring} = 2\pi\sqrt{\frac{m}{k}}$
- $T_{pendulum} = 2\pi\sqrt{\frac{L}{g}}$

Frequency (Hertz/hz) : number of oscillations in a second [cycles / 1 second]
- $f = 1 / T$
- $f_{mass\text{-}spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
- $f_{pendulum} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

Amplitude (s) : maximum distance from the equilibrium position in either direction

Restoring Force (N) : force that restores motion toward equilibrium
- For mass-spring system, restoring force is spring force (F_s)
- For pendulum system, restoring force is force of gravity parallel or mgsinθ (F_g ||)

Hooke's Law

$F_s = -k\Delta x$

k = spring constant [N/m]

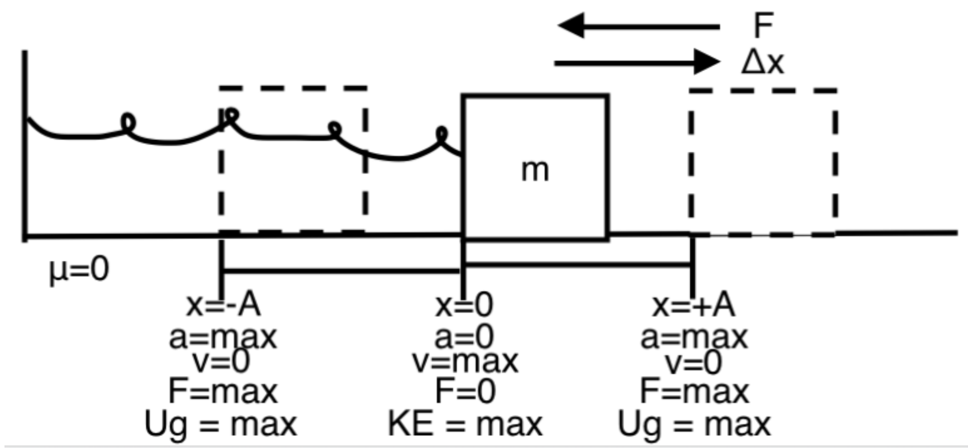
Δx = displacement from equilibrium [m]

F_s varies linearly with Δx
- Doubling displacement = double spring force

Negative k means that the Force (F_s) goes in opposite direction of displacement (Δx) ALWAYS
- Δx + F_s -

- Δx - F_s +

Mass-Spring System



$KE = PE$ @ $A(\frac{\sqrt{2}}{2})$

Total = KE + U

Total = U + U

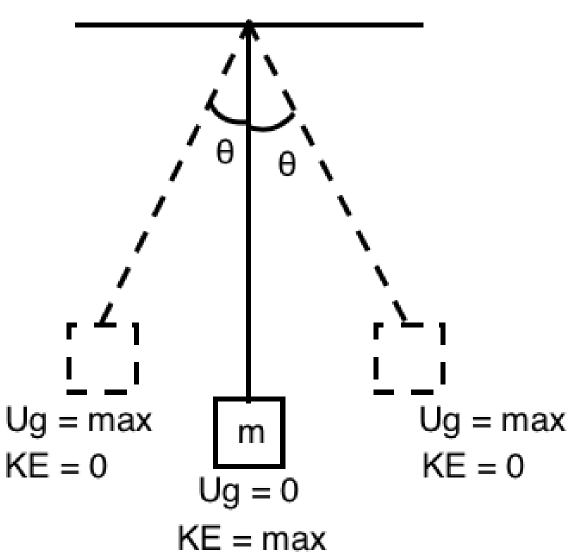
$\frac{1}{2} kA^2 = 2(\frac{1}{2}k\Delta x^2)$

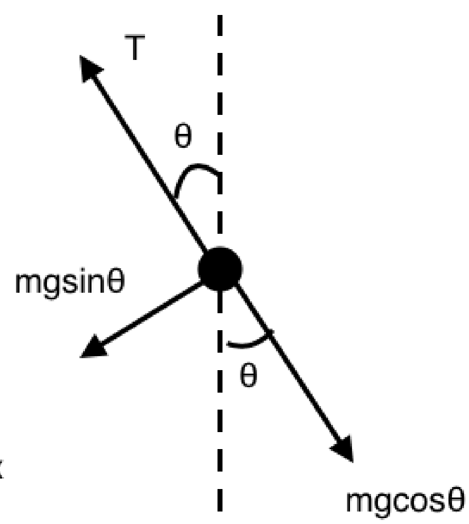
$\frac{1}{2} kA^2 = k\Delta x^2$

$\frac{1}{2} A^2 = \Delta x^2$

$\Delta x = \sqrt{\frac{1}{2}}A$

Pendulum System





Restorative force = F = mgsinθ

- No force at equilibrium because $\theta = 0$

PE = KE @ $\Delta y/2$

Only SHM with small angle approximation so that $\sin\theta = \theta$

Mass-Spring / Pendulum Summary

Mass-Spring	Pendulum
Hooke's Law: $F_s = -k\Delta x$ F_s (restorative force) varies linearly with Δx SHM ✓	$F = -mg\sin\theta$ (restorative force) F doesn't vary linearly with θ F only varies linearly with θ if θ is small (<10˚) measured in radians (Small angle approximation), achieved by restricting domain $F = -mg\theta$ SHM ✓

Period and Frequency of Simple Harmonic Oscillation

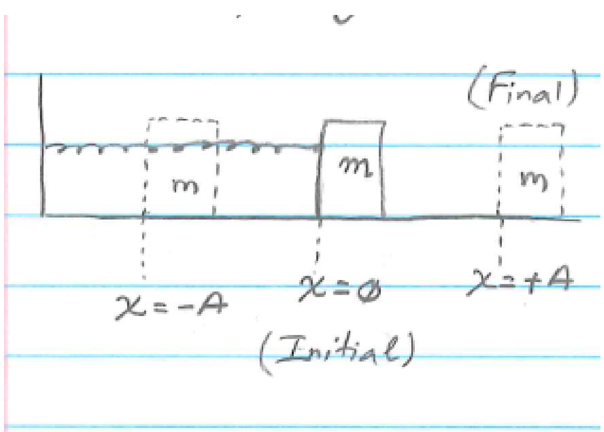
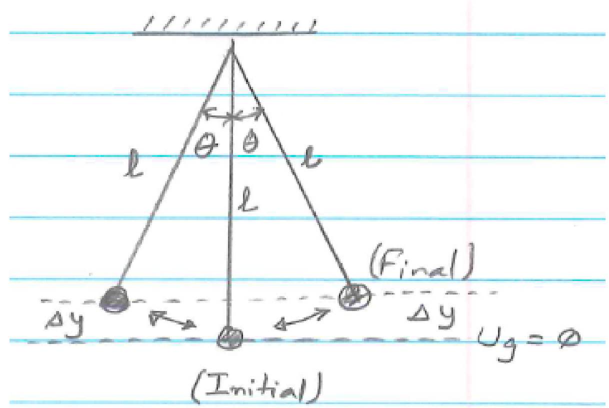
Mass-Spring	Pendulum
$T_{\text{mass-spring}} = 2\pi\sqrt{\frac{m}{k}}$ [sec/cycle] $f_{\text{mass-spring}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ [cycle/sec] m = mass (kg) k = spring constant (N/m)	$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}$ [sec/cycle] $f_{\text{pendulum}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ [cycle/sec] L = pendulum's length (m) g = acceleration due to gravity (m/s^2)

Proportional Reasoning with Frequency and Period

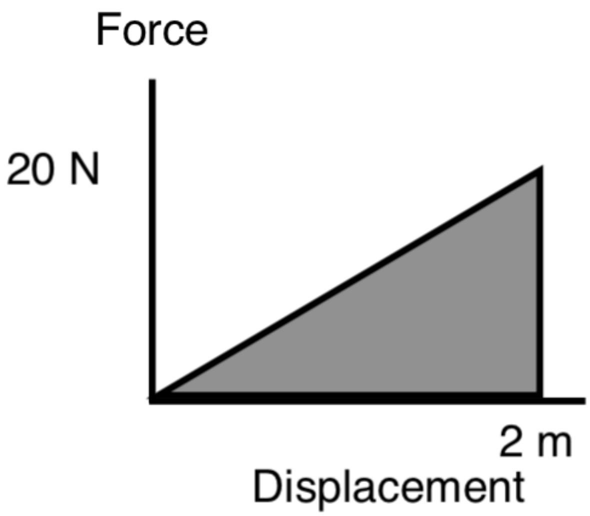
Double Period of Mass-Spring System and Pendulum 1. How to change mass? 4m 2. How to change length? 4L	
Method 1 (Using own numbers) Longer, easier, more room for error	Method 2 (Proportions and underlines) Shorter, harder (requires understanding), less room for error
$T_{\text{mass-spring}} = 2\pi\sqrt{\frac{m}{k}}$ m = 8 kg k = 2 N/m $T_{\text{mass-spring}} = 2\pi\sqrt{\frac{8}{2}}$ $T_{\text{mass-spring}} = 2\pi(2) = 4\pi =$ <u>12.6 seconds</u> $2T = 8\pi = 25.2$ seconds	MANIPULATING NUMERATOR (m or L) $T_{\text{mass-spring}} = 2\pi\sqrt{\frac{m}{k}}$ $T \propto \sqrt{m}$ (T is proportional to square root of m) $2T \propto \sqrt{\_m}$ $2 = \sqrt{x}$ (x is what goes in underline) $4 = x$ $2T \propto \sqrt{4m}$ $2 = \sqrt{4}$ which is true, proportion is correct

$2T_{\text{mass-spring}} = 2\pi\sqrt{\frac{m}{2}}$ $8\pi = 2\pi\sqrt{\frac{m}{2}}$ $4 = \sqrt{\frac{m}{2}}$ $16 = \frac{m}{2}$ m = 32 kg 32 kg compared to 8 kg is four times	Ask yourself what needs to go in the underline so that the proportion or equation is true, "square root of WHAT makes 2?" MANIPULATING DENOMINATOR (k or g) $T_{\text{mass-spring}} = 2\pi\sqrt{\frac{m}{k}}$ $T \propto \sqrt{\frac{1}{k}}$ (T is proportional to square root of 1/k) $2T \propto \sqrt{\frac{1}{\_k}}$ $2 = \sqrt{1/x}$ (x is what goes in underline) $4 = 1/x$ $x = ¼$ $2T \propto \sqrt{\frac{1}{\frac{1}{4}k}}$ $2 = \sqrt{\frac{1}{\frac{1}{4}}}$ which is true, proportion is correct

Energy Conservation and Work in SHM (Mass-Spring and Pendulum)

Mass-Spring	Pendulum
	
$\Delta E_{mech} = 0$ $U_{So} + K_o = U_{Sf} + K_f$ $K_o = U_{Sf}$ $½ mV_{max}^2 = ½kA^2$ $V_{max}^2 = \frac{kA^2}{m}$	$\Delta E_{mech} = 0$ $U_{go} + K_o = U_{gf} + K_f$ $K_o = U_{gf}$ $½ mV_{max}^2 = mg\Delta y$ $½ V_{max}^2 = g\Delta y$ $\Delta y = L(1 - \cos\theta)$ $½ V_{max}^2 = gL(1 - \cos\theta)$

$\mathbf{V_{max}} = \sqrt{\frac{kA^2}{m}}$ Total = KE + U Total = ½ mV^2 + ½$k\Delta x^2$ **Total = ½kA^2** **Total = ½ mV_{max}^2**	$\mathbf{V_{max}} = \sqrt{2gL(1 - cos\theta)}$



W = (F * Δx) / 2
F = kΔx
W = (kΔx * Δx)/2 = ½ $k\Delta x^2$ → Potential Spring Energy
All the work done by the force is stored as potential energy in the spring

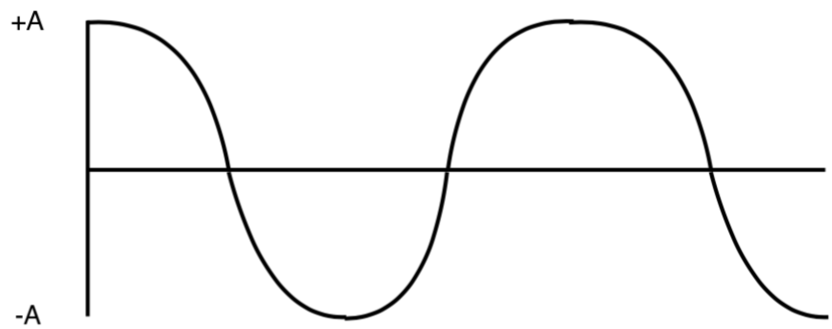
W = F*Δx
W = mgΔx → Gravitational Potential Energy
All the work done by the force is stored as gravitational potential energy

Position, Velocity, and Acceleration for Oscillatory Movement

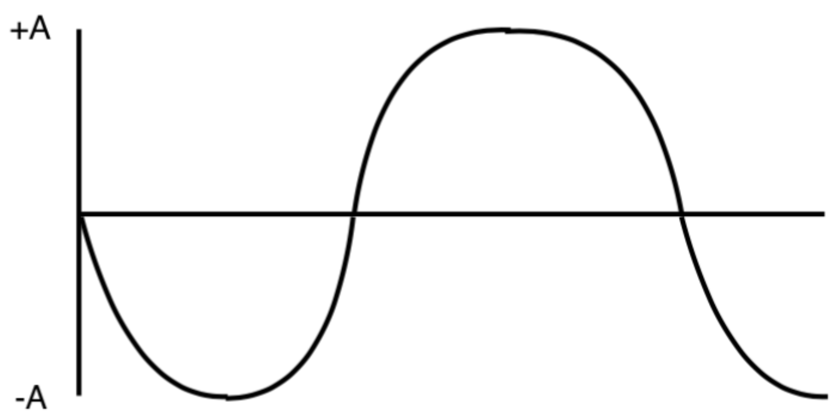
Position: x(t) = Acos(2πft)
Position x(t) = Acos($\frac{2\pi}{T}t$)
Velocity: v(t) = -A2πfsin(2πft) == -Asin(2πft) * (2πf)
v_{max} = -A2πf v_{max} occurs at equilibrium or t = 1 / (4f) = T / 4, T/4 at equilibrium
Acceleration: a(t) = -A4$\pi^2 f^2$cos(2πft) == -Acos(2πft) * (2πf)(2πf)
a_{max} = -A4$\pi^2 f^2$ a_{max} occurs at amplitude or t = 0
A = amplitude
f = frequency
t = time

T = period

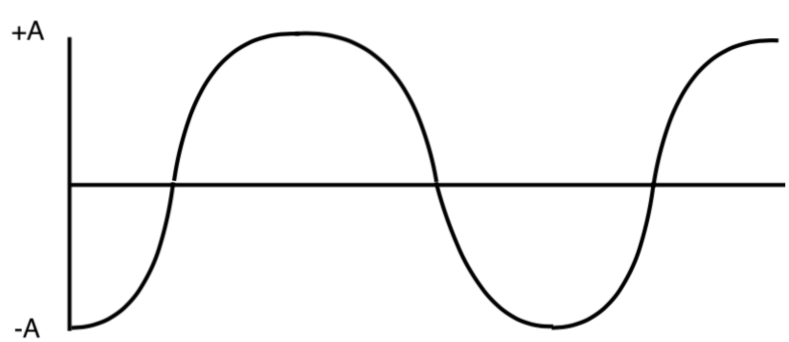
Position: $x(t) = A\cos(2\pi ft)$



Velocity: $v(t) = -A2\pi f\sin(2\pi ft)$



Acceleration: $a(t) = -A4\pi^2f^2\cos(2\pi ft)$



$a_{max} = 4\pi^2 f^2 * A$ $F_s = k\Delta x$ $F_s = ma_{max}$

$ma_{max} = kA$ $f_{\text{mass-spring}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$a_{max} = k/m * A$ $f^2 = \frac{1}{4\pi^2} * \frac{k}{m}$

$4\pi^2 f^2 * A = k/m * A$

$4\pi^2 f^2 = k/m$ $4\pi^2 f^2 = k/m$ TRUE, concept agrees with math

$V_{max} = 2\pi f * A$

$KE_{max} = U_{smax}$ $f_{\text{mass-spring}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\frac{1}{2}mV_{max}^2 = \frac{1}{2}kA^2$ $2\pi f = \sqrt{\frac{k}{m}}$

$V_{max} = \sqrt{\frac{kA^2}{m}} = \sqrt{\frac{k}{m}} * A$

TIPS ON HOW TO SOLVE PROBLEMS:

1. Amplitude does NOT influence period or frequency
    a. Amplitude isn't required in the equations for period or frequency
2. How position graph relates to velocity and acceleration, follows the same pattern
    a. sin → cos → -sin → -cos (Calculus Trig Derivatives)
    b. Position (sin) → Velocity (cos) → Acceleration (-sin)
    c. Position (cos) → Velocity (-sin) → Acceleration (-cos)
3. ½ kA^2 is total amount of energy in the system, all energy is potential at amplitude
4. Proportional reasoning: make sure you factor in the square root when manipulating k, m, or l in order to get desired period or frequency
    a. ONLY MULTIPLY COEFFICIENTS OF MODIFIED VARIABLES
        i. Example: ½ in kinetic energy isn't factored in when doing proportional reasoning
            1. Quadruple KE = 4KE = ½ $m(2v)^2$
                a. $4 = 2^2$
5. If there is displacement present, you are NOT at equilibrium and there is BOTH potential and kinetic energy at that position
6. If there is velocity present, you are NOT at amplitude and there is BOTH potential and kinetic energy at that position
7. Total energy = kinetic + potential
    a. $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}k\Delta x^2$
    b. $\frac{1}{2}mv_{max}^2 = \frac{1}{2}mv^2 + \frac{1}{2}k\Delta x^2$

UNIT 8: Rotational Motion

Rotational Kinematics: Explains how the motions are rotating or moving
Rotational Dynamics: Explains causes of motions, answers "why" motion occurred

Fundamentals of Unit 8 Physics (Rotational Kinematics)

Rotational motion is different from circular motion in the sense that objects not just moving in a circle with constant speed but they can speed up or slow down their rotations

Rotational/Angular Quantities

- Counterclockwise = positive
- Clockwise = negative
- Units go from latin to greek letters

Angular Position : θ (theta) [radians]

- The position of an object in a circle radius r

Angular Displacement : Δθ (theta) [radians]

- Change in position of an object on a circle radius r
- $\Delta\theta = \theta_f - \theta_o$
- Displacement is measured by how far it has gone on the circle, unlike linear
    - One full circle is 2π radians of angular displacement and 0 meters in linear displacement
- Connected to tangential displacement

Angular Velocity : ω (omega) [radians/seconds]

- Rate of change of angular position
- $\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_o}{t_f - t_o} = \frac{\theta_f - \theta_o}{t}$ because $t_o = 0$
- Connected to tangential velocity

Angular Acceleration : α (alpha) [radians/seconds^2]

- Rate of change of angular velocity
- $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_o}{t_f - t_o} = \frac{\omega_f - \omega_o}{t}$ because $t_o = 0$
- Connected to tangential acceleration
- Slowing down = ω and α different signs
- Speeding up = ω and α same signs

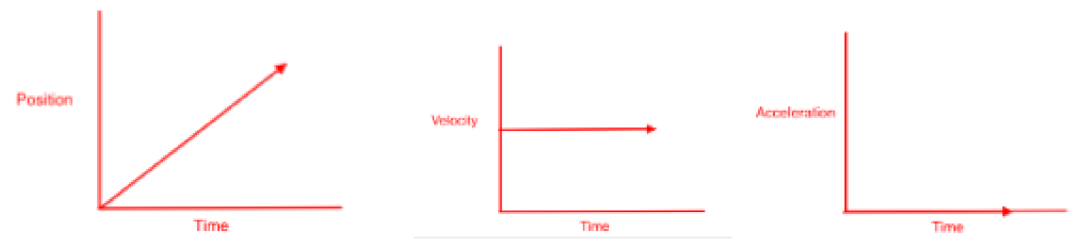
Angular Kinematics (x = θ, v = ω, a = α) (α is constant)

1. $\omega_f = \omega_o + \alpha t$ Excludes $\Delta\theta$
2. $\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$ Excludes ω_f

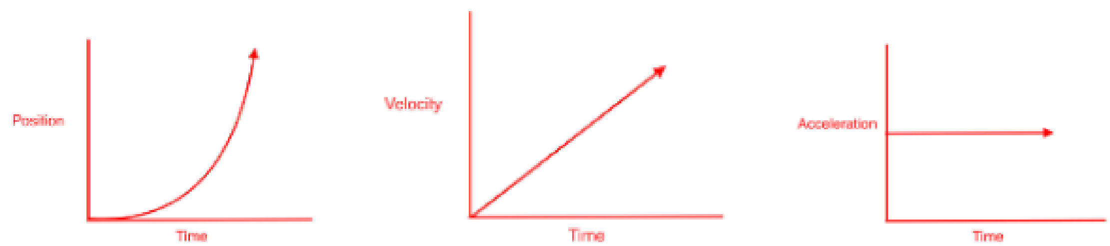
3. $\omega_f^2 = \omega_o^2 + 2\alpha\Delta\theta$ Excludes t

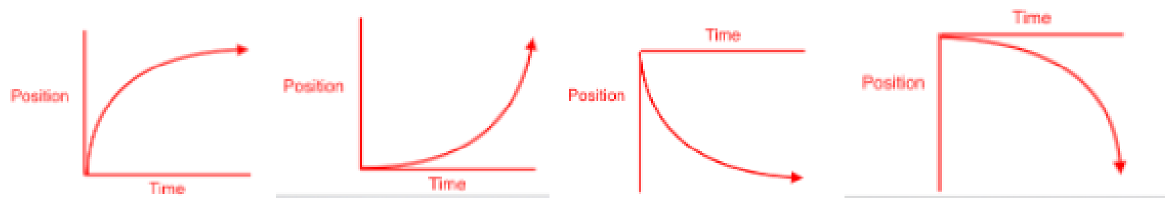
- Same exact graphs as linear motion

Uniform Motion (UM) : motion where objects move with non zero constant velocity



Uniform Accelerated Motion (UAM) : motion where objects move with non zero constant acceleration





Slowing down in positive direction | Speeding up in positive direction | Slowing down in negative direction | Speeding up in negative direction

Linear vs Angular Quantity Table

- Transitioning from linear quantities to angular quantities
- Linear = Tangential
- Angular = Rotational
- Displacement → Velocity → Acceleration, divide by t

Quantity	Linear	Angular	Transition Equation
Position	x	θ	$x = r\theta$
Displacement	Δx	$\Delta\theta$	$\Delta x = r\Delta\theta$

Velocity	v	ω	$v = r\omega$
Acceleration	a_t	α	$a_t = r\alpha$
Mass	m	I (Moment of Inertia)	$m = \frac{I}{r^2}$
Kinetic Energy	K	K_r	$K_r = \frac{1}{2}I\omega^2$
Force	F	τ	$\tau = rF\sin\theta$ $\tau_{net} = I\alpha$ $\tau = Fr_\perp$
Momentum	p	L	$L = I\omega$ $L = mvr\sin\theta$ $L = mvr_\perp$ $L = rp\sin\theta$ $L = pr_\perp$

Linear vs Angular Physics Toolbox Table

Tool	Linear	Angular
Newton's Laws	N1L: $\Sigma F = 0$ N2L: $\Sigma F = ma$	N1LR: $\Sigma\tau = 0$ N2LR: $\Sigma\tau = I\alpha$
Work-KE	$W = \Delta KE$ $W = \frac{1}{2}mv^2_f - \frac{1}{2}mv^2_o$ $W = F\Delta x$	$W = \Delta KE_R$ $W = \frac{1}{2}I\omega^2_f - \frac{1}{2}I\omega^2_o$ $W = \tau\Delta\theta$
Conservation of Energy	$\Delta E = 0$ $U_{go} + K_o = U_{gf} + K_f$ $U_g = mg\Delta y = mgh$ $K_o = \frac{1}{2}mv^2$	$\Delta E = 0$ $U_{go} + K_o + K_{Ro} = U_{gf} + K_f + K_{Rf}$ $U_g = mg\Delta y = mgh$ $K_o = \frac{1}{2}mv^2$ $K_r = \frac{1}{2}I\omega^2$
Impulse-Momentum	$F * \Delta t = \Delta p$ $p = mv$	$\tau * \Delta t = \Delta L$ $L = I\omega$

	$\Delta p = m\Delta v$ $F * \Delta t = m\Delta v$	$\Delta L = I\Delta\omega$ $\tau * \Delta t = I\Delta\omega$
Conservation of Momentum	$\Delta p = 0 \quad \Sigma F_{ext} = 0$ $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$	$\Delta L = 0 \quad \Sigma\tau_{ext} = 0$ $I_1\omega_1 + I_2\omega_2 = I_1\omega_1' + I_2\omega_2'$

Arc Length

Arc length is equal to linear displacement

- S (Arc length) = $r\Delta\theta$
- Circumference = $r(2\pi)$

Centripetal Acceleration and Tangential Acceleration

- Centripetal acceleration = radial acceleration
    - Toward center of circle, perpendicular
    - $a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$
- Tangential Acceleration
    - Tangent to circle, parallel
    - $a_t = r\alpha$
- $a_{total} = \sqrt{a_t^2 + a_c^2} = r\sqrt{\omega^4 + \alpha^2}$ (in terms of angular quantities)

Moment of Inertia, Rotational Inertia, or Moment

Mathematical degree of difficulty in rotating an object about a preferred axis

- Big moment of inertia = harder to move/rotate
- Mass closer to middle = easier to stop : smaller r = smaller I
- Mass farther from middle = harder to stop : greater r = greater I

Inertia's relationship to angular and linear velocity is inversely proportional

- Angular and linear velocity increases as moment of inertia decreases and vice versa
- $v = r\omega$

Moment of Inertia serves in the same capacity as a mass in translational motion, therefore in a sense, moment of inertia is an "angular mass"

- $I \propto m$
- $I \propto r^2$
- $I = mr^2$
- $m = I / r^2$

Calculating Inertia:

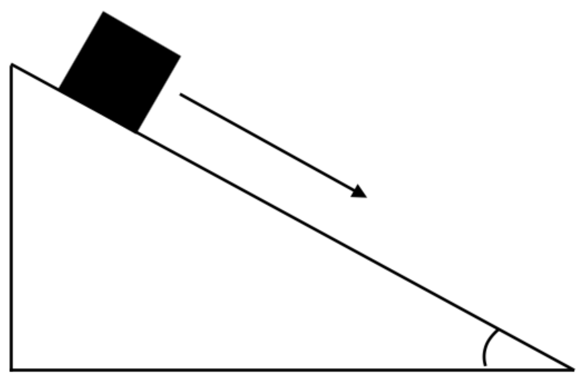
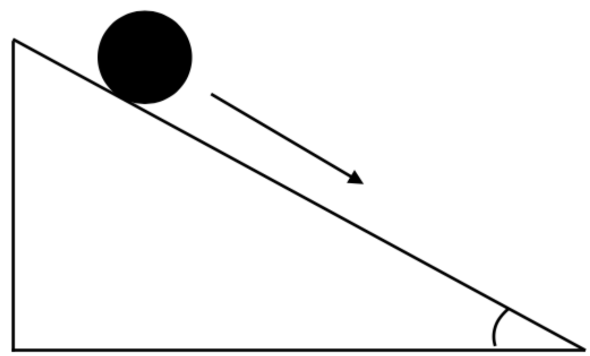
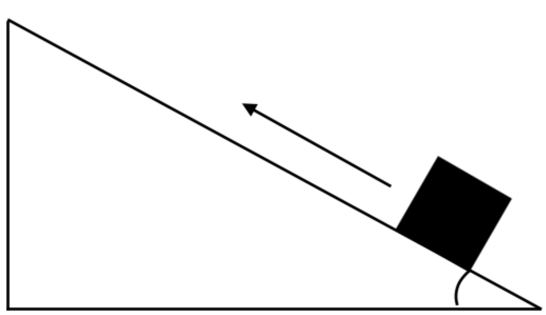
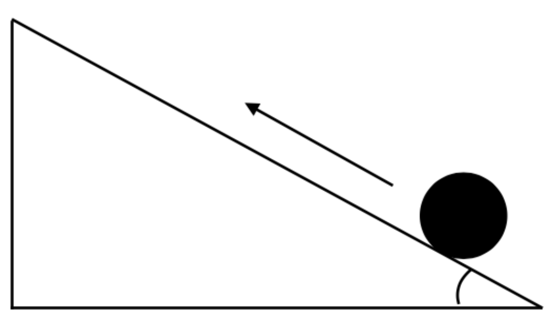
- Need to be calculated by axis
- $I_x = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$
- $I_y = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$
- $I_z = I_x + I_y$

Calculating Inertia for shapes/objects:
- I_{cm} = Moment of Inertia at center of mass
- $I_{cm} = fmr^2$
    - f = shape factor
- Smaller moment = faster

Inertias for common objects:

Ring/Hoop/Thin Cylindrical Shell = MR^2	$f = 1$
Cylinder/Solid Cylinder/Disk = $\frac{1}{2}MR^2$	$f = \frac{1}{2}$
Solid **Sphere** = $\frac{2}{5}MR^2$	$f = \frac{2}{5}$
Shell/Thin Spherical Shell = $\frac{2}{3}MR^2$	$f = \frac{2}{3}$

<u>Rotational Kinetic Energy</u>

Linear	Rotational
$K = \frac{1}{2} mv^2$	$K_r = \frac{1}{2}I\omega^2$
 $U_g \rightarrow KE$ $mgh \rightarrow \frac{1}{2}mv^2$ Faster than ball at bottom of ramp (Bigger v)	 $U_g \rightarrow KE + KE_r$ $mgh \rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ Slower than box at bottom of ramp (Smaller v)
 $KE \rightarrow U_g$ $\frac{1}{2}mv^2 \rightarrow mgh$ Box won't travel as far up the ramp compared to ball (Lower h)	 $KE + KE_r \rightarrow U_g$ $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \rightarrow mgh$ Ball will travel farther up the ramp compared to ball (Higher h)

	More height gained due to rotational kinetic energy

- K_r in terms of linear variables
    - I in terms of m and r
        - $I = fmr^2$
    - ω in terms of v and r
        - ω = v / r

Fundamentals of Unit 8 Physics (Rotational Dynamics)

Answers why motion is happening
Utilizes Newton's Laws of Motion for explaining causes of motion (N1L and N2L)

Newton's 1st Law Rotational (N1LR) : objects at rest, stay at rest, and objects in rotational motion will continue to rotate with constant angular velocity unless an unbalanced torque is applied

Newton's 2nd Law Rotational (N2LR) : Net torque causes the object to angularly accelerate
- $\tau_{net} = \Sigma\tau = I\alpha$
- Similar to F = ma
    - m = I (moment)
    - a = α (angular acceleration)

Angular Force = Torque : τ (tau) [Newtons * meters]
- τ = rFsinθ
- τ = F * "lever arm"
- In order to change angular velocity of an object, must apply torque
- Depends on
    - Magnitude of Force (F)
        - Force increase, torque increases
    - Direction of Force (sinθ)
        - θ closer to perpendicular (π/2 or 90˚), torque increases
        - Angle between r and Force
    - Location of Force (r)
        - r increase, torque increases
        - Distance between force and axis of rotation

Static Equilibrium

Conditions for static equilibrium:
1. ΣF = 0 N1L
2. Στ = 0 N1LR

Not moving or moving with constant velocity
Not rotating or rotating with constant angular velocity

Lever Arm

"Lever arm" is the "effective" distance from the axis of rotation and where the force is applied

“Lever arm” = rsinθ

Find lever arm by extending force vector to infinity and connecting it to pivot spot so that they intersect perpendicularly

Bridge Problem:

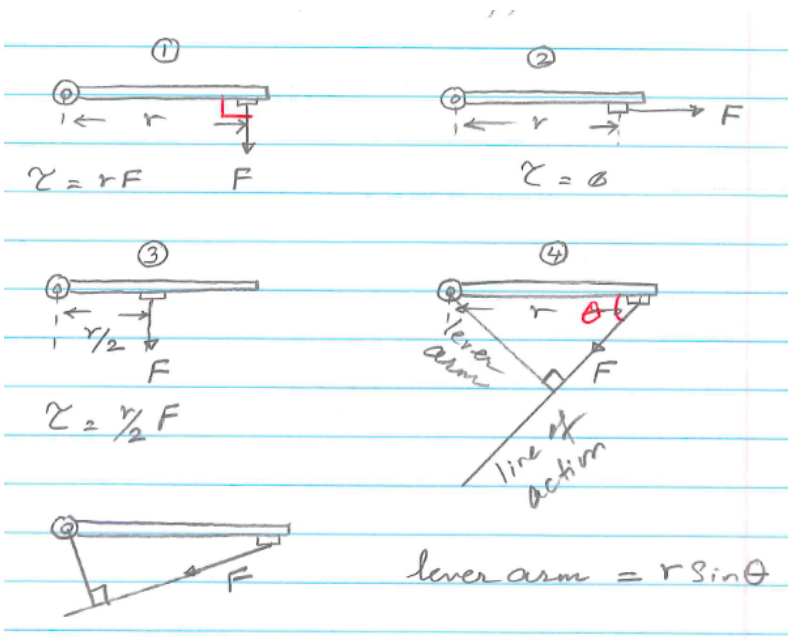
- Pretty much from the pivot point to the force if all forces are perpendicular

Ladder Problem:

- Need to extend force vectors to infinity to see where pivot intersects perpendicularly

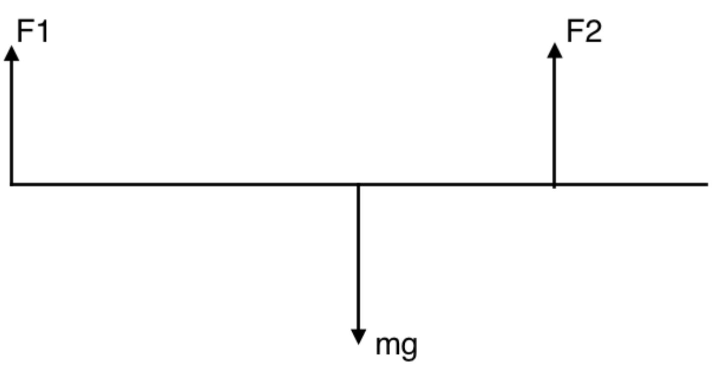
Iron Worker on Beam Problem:

- Very similar to bridge, need to split forces at angle to force in x and y direction



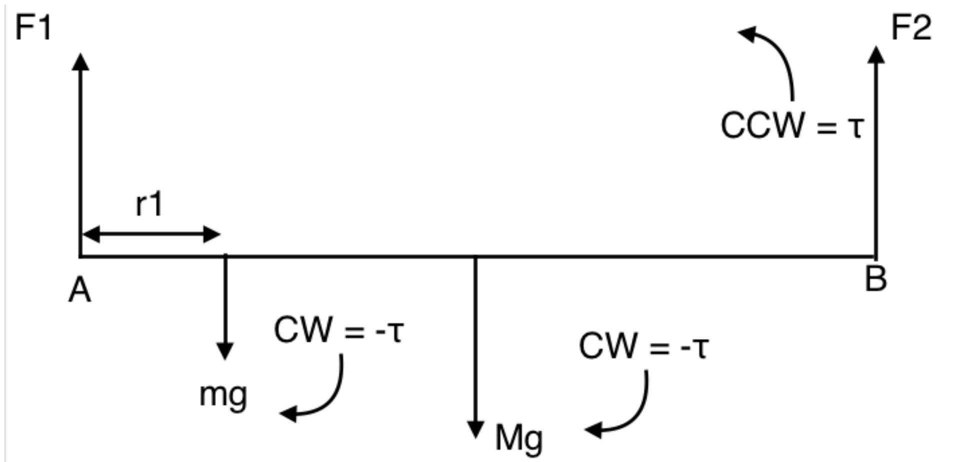
Force Diagram

Shows the location of the forces in relation to other forces, unlike free-body-diagram



Application of Torque

1. Bridge Problem

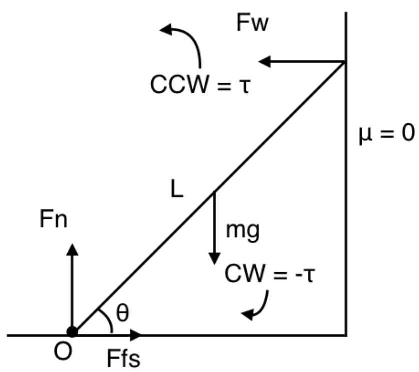


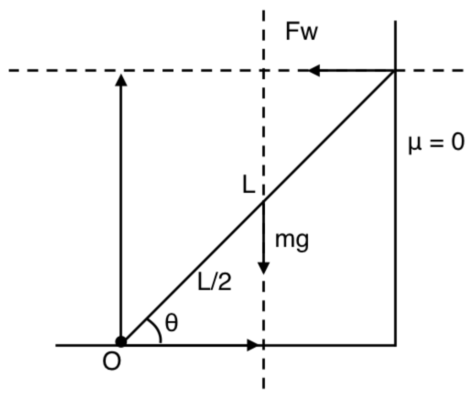
$\Sigma F = 0$ N1L
$F_1 + F_2 - mg - Mg = 0$
$F_1 + F_2 = mg + Mg$
Solve for F_2
Then solve for F_1

$\Sigma\tau_A = 0$ (Pivot = A) N1LR
$\tau_{F2} - \tau_{mg} - \tau_{Mg} = 0$
$(L)(F_2) - (r_1)(mg) - (L/2)(Mg) = 0$

$\tau = rF\sin\theta$

2. Ladder Problem





$\Sigma F_y = 0$ N1L
$F_N - mg = 0$
$F_N = mg$

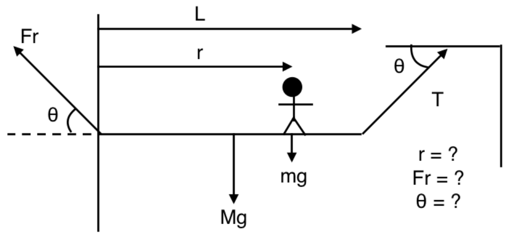
$\Sigma F_x = 0$ N1L
$F_w - F_s = 0$
$F_w = F_s$
$F_w = \mu_s F_N$
$F_w = \mu_s mg$

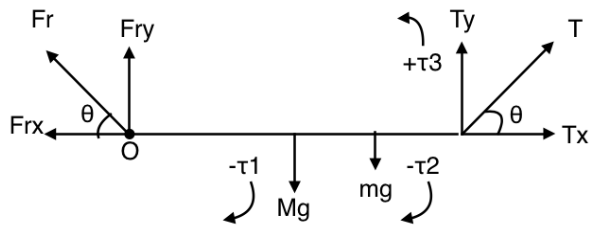
$\Sigma\tau_o = 0$ (Pivot = O) N1LR
$\tau_w - \tau_{mg} = 0$
$(L\sin\theta)(F_w) - (L/2\ \cos\theta)(mg) = 0$
$(L\sin\theta)(\mu_s mg) - (L/2\ \cos\theta)(mg) = 0$
$(\mu_s Lmg * \sin\theta) - (Lmg/2 * \cos\theta) = 0$
$\mu_s Lmg * \sin\theta = Lmg/2 * \cos\theta$
$\mu_s \sin\theta = \frac{1}{2}\cos\theta$
$\mu_s \tan\theta = \frac{1}{2}$
$\tan\theta = 1/(2\mu_s)$
$\theta = \tan^{-1}(1/(2\mu_s))$

$\tau = rF\sin\theta$

Minimum angle that the ladder makes with the floor so it doesn't slip is $\tan^{-1}(1/(2\mu_s))$.
- Depends only on coefficient of static friction (μ_s)

3. Iron Worker on the Beam Problem





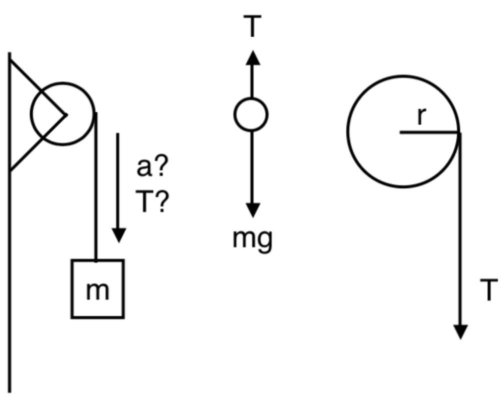
$\Sigma F_x = 0$ N1L
$T_X - F_{Rx} = 0$
$T_X = F_{Rx}$
$F_{Rx} = T\cos\theta$

$\Sigma F_y = 0$ N1L
$F_{Ry} + T_y - Mg - mg = 0$
$F_{Ry} + T\sin\theta - Mg - mg = 0$
$F_{Ry} = Mg + mg - T\sin\theta$

$\Sigma \tau_o = 0$ (Pivot = O) N1LR $\tau = rF\sin\theta$
$\tau_3 - \tau_1 - \tau_2 = 0$
$(L)(T_y) - (L/2)(Mg) - (r)(mg) = 0$
r = farthest distance person can walk without beam breaking

$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2}$
$\theta = \tan^{-1}(F_{Ry} / F_{Rx})$

Spool Disk

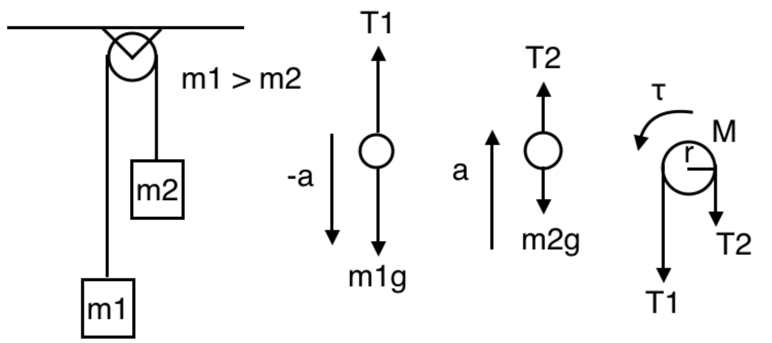


$\Sigma F = ma$
$T - mg = -ma$

$\Sigma\tau = I\alpha$
$rT = I\alpha$
$rT = (\frac{1}{2}Mr^2)(a / r)$
$T = \frac{1}{2}Ma$

Substitute T and find expression for a
$\frac{1}{2}Ma - mg = -ma$
$\frac{1}{2}Ma + ma = mg$
$a(\frac{1}{2}M + m) = mg$
$a = (m / \frac{1}{2}M + m)g$ $T = \frac{1}{2}M(m / \frac{1}{2}M + m)g$

Counter-weight Pulley

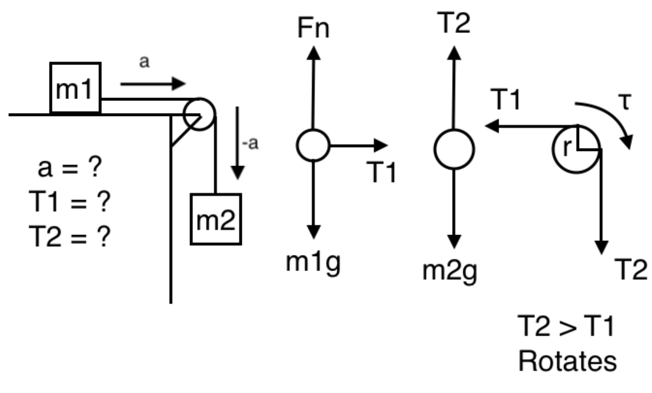


Mass 1:
$\Sigma F = ma$
$T_1 - m_1g = -m_1a$
$T_1 = m_1g - m_1a$

Mass 2:
$\Sigma F = ma$
$T_2 - m_2g = m_2a$
$T_2 = m_2g + m_2a$

Disk: (Bigger term "T_1" first)
$\Sigma\tau = I\alpha$
$rT_1 - rT_2 = (\frac{1}{2}Mr^2)(a/r)$
$T_1 - T_2 = \frac{1}{2}Ma$
$(m_1g - m_1a) - (m_2g + m_2a) = \frac{1}{2}Ma$
$m_1g - m_1a - m_2g - m_2a = \frac{1}{2}Ma$
$m_1g - m_2g = m_1a + m_2a + \frac{1}{2}Ma$
$m_1g - m_2g = a(m_1 + m_2 + \frac{1}{2}M)$
$a = ((m_1 - m_2) / (m_1 + m_2 + \frac{1}{2}M))g$
Find T_1 and T_2 by subbing in for "a"

Atwood's Machine with a Massive Pulley



Mass 1:
$\Sigma F = ma$
$T_1 = m_1a$

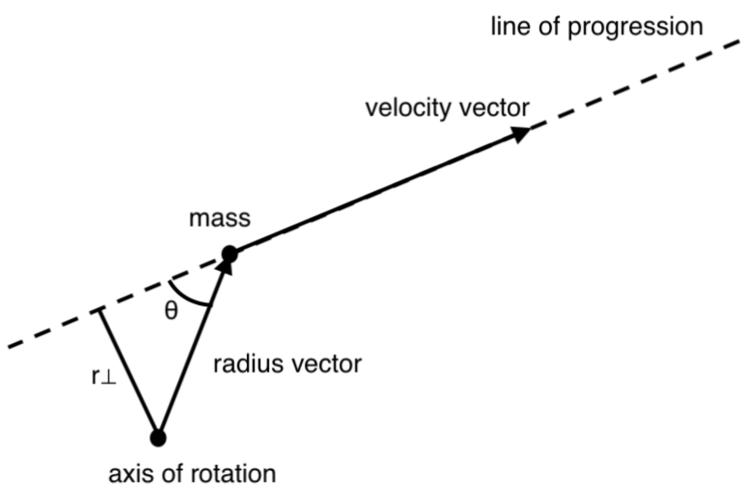
Mass 2:
$\Sigma F = ma$
$T_2 - m_2 g = -m_2a$
$T_2 = m_2g - m_2a$

Disk:
$\Sigma\tau = I\alpha$
$rT_2 - rT_1 = (\frac{1}{2}Mr^2)(a/r)$
$T_2 - T_1 = \frac{1}{2}Ma$
$m_2g - m_2a - m_1a = \frac{1}{2}Ma$
$\frac{1}{2}Ma + m_2a + m_1a = m_2g$
$a(\frac{1}{2}M + m_2 + m_1) = m_2g$

$a = (m_2 / (\frac{1}{2}M + m_2 + m_1))g$
Find T_1 and T_2 by subbing in for "a"

Angular Momentum

L is a vector quantity
$L = I\omega$
$L = mvr\sin\theta$
$L = mv(r_\perp)$ $r_\perp = r\sin\theta$ = lever arm θ = angle between radius vector and line of progression
$L = rp\sin\theta$ p = linear momentum

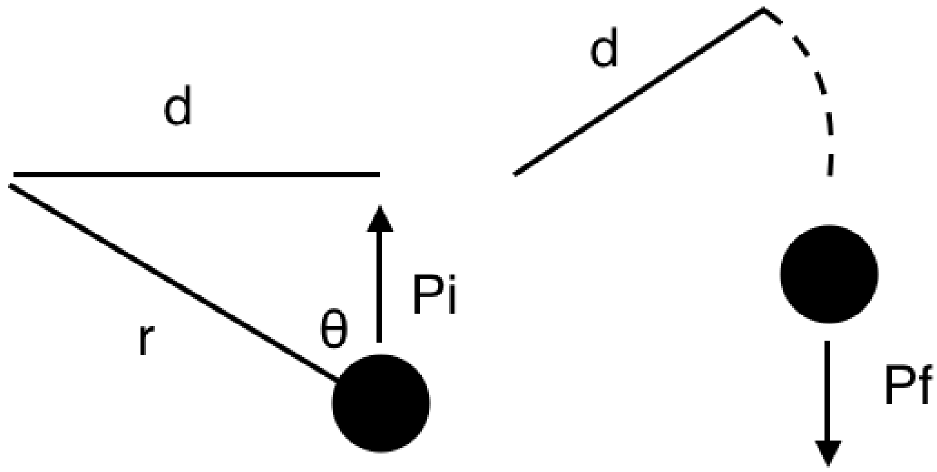


Conservation of Angular Momentum

Linear: $\Delta p = 0$ $\Sigma F_{Ext} = 0$ (No resistive forces = air resistance, friciton, etc.)
Angular: $\Delta L = 0$ $\Sigma \tau_{Ext} = 0$ (No resistive torques = friction, etc.)
$L_o = L_f$
$I_o\omega_o = L_f\omega_f$

Example: Khan holding spinning wheel on chair, flipping wheel upside-down causes Khan to spin
$\Delta L = 0$ $\Sigma \tau_{Ext} = 0$
W = wheel, K = Khan
$L_o = L_f$
$L_{wo} + L_{ko} = L_{wf} + L_{kf}$
$L_{wo} = L_{wf} + L_{kf}$
$L_{wo} = -L_{wo} + L_{kf}$
$L_{kf} = 2L_{wo}$
This is why Mr. Khan begins spinning

Example: Ball hitting rod, what is the rod's angular momentum if the ball bounces back?



$\Delta L = 0$ $\Sigma\tau_{Ext} = 0$

R = Rod, B = Ball

$L_o = L_f$

$L_{Bo} + L_{Ro} = L_{Bf} + L_{Rf}$

$L_{Bo} = L_{Bf} + L_{Rf}$ ($L_{Ro} = 0$ because rod is at rest)

$mvr\sin\theta = mvr\sin\theta + L_{Rf}$

$P_o r\sin\theta = -P_f r\sin\theta + L_{Rf}$ ($P = mv$, $-P_f$ is final momentum of the ball, P_o is initial momentum of the ball)

(P_f is negative because the ball is now going in the opposite direction)

$P_o r\sin\theta + P_f r\sin\theta = L_{Rf}$

$r\sin\theta(P_o + P_f) = L_{Rf}$

$d(P_o + P_f) = L_{Rf}$ ($d = r\sin\theta$ because $\sin\theta = d/r$)

<u>TIPS ON HOW TO SOLVE PROBLEMS:</u>

1. Choose your pivot that eliminates the most forces, the point with most forces coming from it
    a. The removes more torque that you have to solve for, removes torque because their “r” is 0
2. CW = negative torque, CCW = positive torque
3. Ladder problem angle so that ladder doesn’t slip:
    a. $\theta = \tan^{-1}(1/(2\mu_s))$
4. Inertia
    a. Big moment of inertia = harder to move/rotate
    b. Mass concentrated closer to middle = easier to stop : smaller r = smaller I
    c. Mass concentrator farther from middle = harder to stop : greater r = greater I
    d. Inertia inversely proportional to angular velocity and linear velocity
    e. Lowest shape factor = larger angular velocity and linear velocity
        i. Sphere → Cylinder → Ring
5. Angular kinematic equations are exactly the same, change variables from latin to greek
    a. $\theta = x$
    b. $\omega = v$
    c. $\alpha = a$

6. Friction ramp (ball rolling):
    a. Going up ramp: $KE + KE_R \rightarrow U_g$
    b. Going down ramp: $U_g \rightarrow KE + KE_R$
7. Non friction ramp (ball slipping)
    a. Going up ramp: $KE \rightarrow U_g$
    b. Going down ramp: $U_g \rightarrow KE$
8. Smaller radius = larger angular velocity
    a. $v = r\omega$
9. Linear velocity doesn't depend on radius or mass
10. Angular velocity INDEPENDENT of radius because it's a measured quantity, like time
11. If comparing dropping blocks on a spool with same torque, the block with lower inertia (smaller r) will have higher angular acceleration and will reach the ground faster
12. Use Linear vs Angular toolbox table to figure out formulas or which equations to use